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Short Communication

A note on: Optimal ordering policy for stock-dependent demand under progressive payment scheme

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ABSTRACT

In a recent paper, Soni and Shah [Soni, H., Shah, N. H. (2008). Optimal ordering policy for stock-dependent demand under progressive payment scheme. European Journal of Operational Research 184(1), 91–100] developed a model to find the optimal ordering policy for a retailer with stock-dependent demand and a supplier offering a progressive payment scheme to the retailer. This note corrects some errors in the formulation of the model of Soni and Shah. It also extends their work by assuming that the credit interest rate of the retailer may exceed the interest rate charged by the supplier. Numerical examples illustrate the benefits of these modifications.

1. Introduction

Recently, Soni and Shah (2008) developed a model to find the optimal ordering policy for a retailer with stock-dependent demand and a supplier that offers a progressive payment scheme to the retailer. The authors assumed that if the retailer settles its balance before time M, the supplier charges no interest to the retailer, whereas in case when the retailer settles its balance between times M and N with $M < N$, the supplier charges an interest rate Ic_1 on the outstanding balance. In the case when the retailer pays after time N, the supplier charges an interest rate Ic_2 with $Ic_2 > Ic_1$. Revenues the retailer receives from sales may be deposited in an interest-bearing account until the account is settled completely, where an interest is earned at the rate of Ie.[1] Soni and Shah (2008) assumed that in case the retailer is not able to settle its unpaid balance at time M (or N), s/he will settle as much of the unpaid balance as possible at these points in time. The work of Soni and Shah (2008) was extended by Teng, Krommyda, Skouri, and Lou (2011) who included additional aspects in the model, such as deterioration, limited capacity and non-zero ending inventory, and by Shah, Patel, and Lou (2011) who considered a variable retailer selling price in addition to the extensions made by Teng et al. (2011).

In developing their model, Soni and Shah (2008) implicitly assumed that the interest rate charged by the supplier in the first credit period, Ic_1, always exceeds the credit interest rate of the retailer, Ie. We note that this is not necessarily the case in practice. Instead, the interest rates charged by the supplier, Ic_1 and Ic_2, and the credit interest rate of the retailer, Ie, usually depend on the investment opportunities of the respective companies. Ie could thus represent the interest rate the retailer could realize by depositing money in an interest-bearing account, but it could also represent the profit that the retailer can gain from other business activities or its opportunity cost of capital (Summers & Wilson, 2002). The same applies to the interest rates charged by the supplier. It is clear that the ratios of Ie to Ic_1 and Ic_2 thus depend on the individual business environments of the supplier and the retailer, and that Ie could possibly exceed Ic_1 and Ic_2. If Ie exceeds Ic_1, for example, it may not be reasonable for the retailer to settle its unpaid balance at time M, as assumed in Soni and Shah (2008). Instead, it would be better to keep the sales revenue in an interest-bearing account or to invest it elsewhere, and to settle the unpaid balance when the interest charged by the supplier exceeds the returns from interest. This note extends the work of Soni and Shah (2008) by explicitly assuming that the case $Ie > Ic_1$ may occur in addition to the other cases studied by the authors. However, the case $Ie > Ic_2$, where the retailer possibly never pays the supplier, is excluded.

Depending on the ratio of the interest rates Ic_1 and Ie and the time when the retailer sells off the entire production lot, ten different cases arise, which are summarized in Table 1. The cases that were not treated by Soni and Shah (2008) will be discussed briefly in the following, and further some errors contained in their work will be corrected. We adopt the assumptions and notations used in Soni and Shah (2008) hereafter, unless it is stated otherwise.

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[1] Thus, we do not consider investment decisions which are not related to the lot sizing problem.

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Table 1
Cases for settling the unpaid balance.

Ratio of T, M and N	Ratio of interest rates	Unpaid balance	Account settled	Treated in subcase
$T \leqslant M$	$Ic_1 \geqslant Ie$	–	M	1.1
$T \leqslant M$	$Ic_1 < Ie$	–	N	1.2
$M < T \leqslant N$	$Ic_1 \geqslant Ie$	$U_1 = 0$	M	2.1
$M < T \leqslant N$	$Ic_1 \geqslant Ie$	$U_1 > 0$	$M + z$	2.2
$M < T \leqslant N$	$Ic_1 < Ie$	–	N	2.3
$T > N$	$Ic_1 \geqslant Ie$	$U_1 = 0$	M	3.1
$T > N$	$Ic_1 \geqslant Ie$	$U_2 = 0$	$M + z$	3.2
$T > N$	$Ic_1 \geqslant Ie$	$U_2 > 0$	$N + z$	3.3
$T > N$	$Ic_1 < Ie$	$U_3 = 0$	N	3.4
$T > N$	$Ic_1 < Ie$	$U_3 > 0$	$N + z$	3.5

2. Modified model

Subcase 1.1. This case is discussed as ‘Case 1’ in Soni and Shah (2008).

Subcase 1.2. For $T \leqslant M$ and $Ie > Ic_1$, the retailer achieves a financial benefit from postponing the refund and investing the sales revenue until time N. Between times M and N, s/he has to pay interest to the supplier. However, due to $Ie > Ic_1$, the interest earned exceeds the interest paid within the specified time period. The interest earned per year can be calculated as:

$$IE_{1,2} = \frac{PIe}{T}\left(\int_0^T R(t)tdt + Q(N-T)\right) = \frac{PIea}{b^2T}(e^{bT}(b(N-T)+1) - bN - 1) \tag{1}$$

The overall interest charged between M and N amounts to:

$$IC_{1,2} = \frac{Ic_1}{T}CQ(N-M) = \frac{CIc_1a}{bT}(e^{bT}-1)(N-M) \tag{2}$$

The total costs are calculated from Eqs. (1) and (2) by considering ordering cost and inventory holding cost in addition:

$$TC_{1,2} = \frac{A}{T} + \frac{ha}{b^2T}(e^{bT} - bT - 1) + \frac{CIc_1a}{bT}(e^{bT}-1)(N-M) - \frac{PIea}{b^2T} \times (e^{bT}(b(N-T)+1) - bN - 1) \tag{3}$$

The optimal solution to Eq. (3) is the solution of the following non-linear equation:

$$\frac{dTC_{1,2}}{dT} = -\frac{A}{T^2} - \frac{ah(e^{bT} - bT - 1)}{b^2T^2} + \frac{ah(e^{bT}-1)}{bT} + \frac{CIc_1a(bTe^{bT} - e^{bT} + 1)(N-M)}{bT^2} + \frac{IePa(e^{bT} - bT - 1)}{bT} + \frac{IePa(e^{bT}(b(N-T)+1) - bN - 1)}{b^2T^2} = 0 \tag{4}$$

which minimizes $TC_{1,2}$ provided that the second derivation with respect to T is

$$\frac{d^2TC_{1,2}}{dT^2} = \frac{2A}{T^3} - \frac{2CIc_1ae^{bT}(N-M)}{T^2} + \frac{2CIc_1a(e^{bT}-1)(N-M)}{bT^3} + \frac{CIc_1abe^{bT}(N-M)}{T} - \frac{2a(e^{bT}-1)}{bT^2} + \frac{hae^{bT}}{T} + \frac{2ha(e^{bT}-bT-1)}{b^2T^3} - \frac{PIea(1-bT)}{T} - \frac{2PIea(e^{bT}(b(N-T)+1) - bN - 1)}{b^2T^3} - \frac{2PIea(e^{bT} - bT - 1)}{T^2b} > 0, \text{ for all } T. \tag{5}$$

Subcase 2.1. In the case when $M < T \leqslant N$ and $Ie \leqslant Ic_1$, the retailer settles as much of the unpaid balance as possible at time M to minimize interest payments. The first subcase assumes that the sum of sales revenue and interest earned by time M is sufficient to settle the unpaid balance, i.e. $U_1 = 0$. The interest earned until time M is formulated as follows (note that this formulation corrects an error in Soni and Shah’s Eq. (3.11)):

$$IE_{2,1} = \frac{PIe}{T}\int_0^M R(t)tdt = \frac{PIea}{b^2T}e^{b(T-M)}(e^{bM} - bM - 1) \tag{6}$$

As the retailer does not have to pay interest to the supplier in this subcase (i.e. $IC_{2,1} = 0$), the total costs amount to:

$$TC_{2,1} = \frac{A}{T} + \frac{ha}{b^2T}(e^{bT} - bT - 1) - \frac{PIea}{b^2T}e^{b(T-M)}(e^{bM} - bM - 1) \tag{7}$$

The optimal solution to Eq. (7) is the solution of the following non-linear equation:

$$\frac{dTC_{2,1}}{dT} = -\frac{A}{T^2} - \frac{ha(e^{bT} - bT - 1)}{b^2T^2} + \frac{ha(e^{bT}-1)}{bT} + \frac{PIea\ e^{b(T-M)}(e^{bM} - bM - 1)}{b^2T^2} = 0 \tag{8}$$

which minimizes $TC_{2,1}$ provided that the second derivation with respect to T is

$$\frac{d^2TC_{2,1}}{dT^2} = \frac{2A}{T^3} - \frac{2ha(e^{bT}-1)}{bT^2} + \frac{ha\ e^{bT}}{T} + \frac{2ha(e^{bT} - bT - 1)}{b^2T^3} - \frac{2PIea\ e^{b(T-M)}(e^{bM} - bM - 1)}{b^2T^3} > 0, \text{ for all } T. \tag{9}$$

Subcase 2.2. In this subcase, the sum of sales revenue and interest earned by time M is not sufficient to settle the balance completely, i.e. $U_1 > 0$. Thus, the retailer has to pay interest on U_1. Interest earned is the same as the one given in Eq. (6). In calculating the unpaid balance U_1, Soni and Shah (2008) assumed that $U_1 = CQ - (PR(M)M + IE_2)$, where $R(t)$ denotes the stock-dependent demand rate. Since the demand rate decreases in t due to a decreasing inventory level, we note that $PR(M)M$ underestimates the sales revenue of the retailer, since $R(M) < R(M - \Delta)$ for $\Delta > 0$. As a consequence, U_1 has to be reformulated as follows:

$$U_1 = CQ - \left(P\int_0^M R(t)\mathrm{d}t + PIe\int_0^M R(t)t\mathrm{d}t\right) \tag{10}$$

Furthermore, the authors mentioned that the “retailer will have to pay interest on un-paid balance [...] at the rate of Ic_1 at time M to the supplier”. However, we note that after the account has been partially settled at time M, the retailer has no money left to pay interest in advance. We therefore modify Soni and Shah’s approach and assume that when $U_1 > 0$ and $Ie \leqslant Ic_1$, the retailer transfers each dollar s/he earns after time M directly to the supplier to minimize