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Probabilistic modeling of multiperiod service levels

Miguel A. Lejeune [1]

Department of Decision Sciences, George Washington University, Washington, DC, United States

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ABSTRACT

This study investigates multiperiod service level (MSL) policies in supply chains facing a stochastic customer demand. The objective of the supply chains is to construct integrated replenishment plans that satisfy strict stockout-oriented performance measures which apply across a multiperiod planning horizon. We formulate the stochastic service level constraints for the fill rate, ready rate, and conditional expected stockout MSL policies. The modeling approach is based on the concept of service level trajectory and provides reformulations of the stochastic planning problems associated with each MSL policy that can be efficiently solved with off-the-shelf optimization solvers. The approach enables the handling of correlated and non-stationary random variables, and is flexible enough to accommodate the implementation of fair service level policies, the assignment of differentiated priority levels per products, or the introduction of response time requirements. We use an earthquake disaster management case study to show the applicability of the approach and derive practical implications about service level policies.

1. Service level types

Shortage management is a key supply chain performance driver. A stockout causes not only a canceled order (and therefore profit loss), but it also reduces the likelihood of receiving future orders from customers whose demand could not be initially satisfied (see Anderson et al. (2006) for a study about the effects of stockouts). Decision-makers with stockout-averse preferences (i.e., aversion to losing potential sales) have a strong interest in strict service level policies (Schweitzer and Cachon, 2000). These policies are particularly needed for supply chains operating in highly competitive, military (Kress, 2002) or humanitarian (Boin et al., 2010; Campbell and Jones, 2011; Rawls and Turnquist, 2010; Lin et al., 2012) environments, those dealing with a few large and dominant customers (Lejeune and Ruszczyński, 2007), those selling high-profit products (Schweitzer and Cachon, 2000), or those in which a shortage would trigger huge costs to set up the production process all over again (Dauzère-Pérès et al., 2007). The *International Journal of Production Economics* has recently devoted a special issue (Doerner et al., 2011) to extreme supply chains in which high reliability levels and the control of the service level performance over a medium-term planning horizon (see also Van Landeghem and Vanmaele, 2002) are indispensable. This study is motivated by such needs and examines the enforcement of stockout-related multiperiod service level (MSL) policies.

We consider multiperiod planning horizons and define an MSL policy with respect to the probability, proportion or amount of shortage occurring across the entire planning horizon. An MSL policy provides a representative measure of the responsiveness of the supply chain across the planning horizon (Kress, 2002). As opposed to an MSL policy, a stagewise (i.e., one-period) service level (SSL) policy measures the probability, proportion or amount of stockout at any single period considered *independently* of the other periods, and is an expected value measure that reflects the steady-state nature of the supply chain. A high probability of not having a shortage in any single periods of the horizon does not necessarily imply the same high probability of not having a shortage across the whole horizon (Pyke and Cohen, 1993).

Service level policies can be modeled with chance (probabilistic) and conditional expectation constraints (Prékopa, 1995). Constraints representing ready rate and fill rate SSL policies take the form of individual (separate) chance constraints, for which a deterministic equivalent can be easily derived. By contrast, constraints enforcing ready rate MSL policies require the computation of multivariate probabilities, and take the form of joint chance constraints, for which it is more challenging to derive equivalent and tractable deterministic formulations (Lejeune and Ruszczyński, 2007; Prékopa, 1995, 2003).

Beyond the distinction between multiperiod and stagewise, service level policies can be differentiated according to the nature of shortages they prevent. In this paper, we consider the *ready rate* service level which represents the *probability* of not having a stockout, and the *fill rate* and the *conditional expected stockout* service levels, which are related to the shortage *quantities*. A ready rate service level policy is favored when customer satisfaction is

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E-mail address: mlejeune@gwu.edu

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primarily driven by the occurrence or not of a stockout, while its magnitude is secondary (Kleijnen and Smits, 2003). A fill rate service level policy is enforced when the focus is on limiting the expected proportion of demand that cannot be satisfied with the available inventory (Kleijnen and Smits, 2003). The conditional expected stockout (Adenso-Diaz, 1996; Rose, 1972) represents the quantity of products that will be short in case of a stockout. It is particularly important to limit this value when it is necessary to order the amount of non-available product from a second source (Adenso-Diaz, 1996). The reader is referred to Hausman (2002), Kleijnen and Smits (2003), and Tempelmeier (2011) for a discussion on the suitability of the various kinds of service levels.

2. Contributions and literature review

A key contribution of this study is that it provides decision makers with a modeling approach enabling them to implement stockout-related MSL policies. An important feature of the approach is its wide applicability. First, it can be applied to the various types (ready rate, fill rate, conditional expected stockout) of MSLs. Second, it is not affected by restrictive (independence) assumptions about the random variables and can handle stationary as well as non-stationary (i.e., seasonal) random demand. Third, it allows for a fine characterization of uncertainty. Indeed, even if the uncertainty is represented with extremely many scenarios, our modeling approach provides formulations that can be solved with off-the-shelf optimization solvers. Fourth, it can be used to enforce various managerial policies (fairness, differentiated service level, etc.).

We formulate new stochastic models for the construction of replenishment plans accounting for the requirements of three types of MSL policies. The models are multi-functional and define the optimal production/acquisition, distribution, and inventory decisions. Next, we propose a reformulation method that can be applied to the stochastic optimization problems associated with each MSL policy. The reformulated problems are deterministic and equivalent to the original stochastic planning problems. While the stochastic formulations could not be handled by optimization solvers in their original form, the reformulated problems can be efficiently solved with standard mathematical programming algorithms. The method rests on the concepts of service level sufficient trajectories which represent a set of sufficient conditions that must be satisfied in order to attain the prescribed MSL policy threshold. In addition to providing a new reformulation for the event-oriented ready rate MSL policy (Lejeune and Ruszczyński, 2007), we also develop a stochastic programming planning approach, formulate service level constraints, and define service level sufficient trajectories for the fill rate and conditional expected stockout MSLs.

In the brief outline of the literature presented here, we point out models which explicitly account for the uncertainty in production–distribution systems. Cardós et al. (2006) calculate the attained cycle service level in an (R,S) periodic review inventory system when the demand is discretely distributed. Lejeune and Ruszczyński (2007) use the p-efficiency concept (Prékopa, 1995; Dentcheva et al., 2000) to design column generation and preprocessing algorithms allowing for the construction of inventory-production plans in which the multiperiod probability of a stockout is upper-bounded (see also Lejeune, 2008). Paschalidis et al. (2004) derive a base-stock production policy for a multistage supply chain that faces a stochastic demand and whose one-period probability of stockout must be below a prescribed level. Additionally, Chen and Vairaktarakis (2005) develop a production–distribution model in which the goods are directly delivered from the producer to the end-customer. A service level policy based on the time at which the products are supplied to the customers is used. Tempelmeier (2007) studies the uncapacitated single-item dynamic lot-sizing problem. Models that minimize the setup and holding costs and that include a service level constraint are presented. Kutanoglu and Lohiya (2008) develop an integrated base-stock inventory-transportation model for a single-echelon, multi-facility service parts logistics system with time-based service level constraints. Nagar and Jain (2008) use multistage stochastic programming for modeling the launching of new products with uncertain demand. The model allows for the adjustment of the production as uncertainties are resolved. The derivation of the optimal safety stock levels needed to achieve predefined fill rate service levels in multistage networks confronted with uncertain demands is studied by Boulaksil et al. (2009). Simulation is employed to solve the planning problem on a rolling horizon basis. We refer the reader to Chen (2010) for a detailed review of production–distribution models, and to Geunes and Pardalos (2003), Mula et al. (2006), Shen (2007), and the references therein for such models in a stochastic context. Tang (2006) presents the main types of supply chain risk and review the practices and quantitative models enabling to cope with them. Stochastic programming models for supply chain management are proposed by Dormer et al. (2005), while an overview of the formulation of stochastic constraints is given by Prékopa (1995, 2003).

Another key contribution of this study stems from the practical insights derived from the application of our approach to a real disaster management problem. The widespread impact of disasters highlights the need to develop planning models and methods allowing for an appropriate response (Rottkemper et al., 2011). Noting the difficulty to develop accurate demand forecasts due to the lack or inconsistent collection of historical data, and the multiple unknowns that humanitarian logisticians have to deal with, Ergun et al. (2011) advocate the development of stochastic planning models accounting for the many sources of uncertainty. Further, Snyder (2006) and Campbell and Jones (2011) insist about the importance to account for a wide range of possible situations and to consider a large number of scenarios in the disaster management and humanitarian relief contexts. The proposed models take an explicit consideration of the randomness of the demand for relief commodities, and can be conveniently used to implement disaster pre-positioning and response policies that satisfy strict service level requirements. Moreover, our model provides formulations that can be solved very efficiently regardless of the number of scenarios used to represent uncertainty. We illustrate the impact and adequacy of MSL versus SSL policies, and how priority levels per commodity, as well as fairness and response time policies, can be effortlessly integrated with the proposed approach. We refer the reader to the special issues of the *International Journal of Production Economics* (Volume 126, Issue 1, 2010) and *OR Spectrum* (Volume 33, Issue 3, 2011) providing for a thorough review of the disaster management literature, and to Altay and Green (2006) and Van Wassenhove and Pedraza Martinez (2012) for a detailed discussion of the contribution of OR techniques to the disaster management field.

In Section 3 of this paper, we define the general formulation of the stochastic replenishment model that enforces an MSL policy. In Section 4, we elaborate on the modeling approach and derive the new stochastic planning models and their deterministic reformulation. Section 5 analyzes the application of the approach to a disaster management supply chain problem. Section 6 offers concluding remarks.

3. Probabilistic replenishment planning model

We develop a replenishment planning model for integrated inventory, production/acquisition, and distribution decision making in a multistage (e.g., suppliers, manufacturers, distributors)