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Intuitionistic fuzzy optimization technique for Pareto optimal solution of manufacturing inventory models with shortages

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ABSTRACT

This paper discusses a manufacturing inventory model with shortages where carrying cost, shortage cost, setup cost and demand quantity are considered as fuzzy numbers. The fuzzy parameters are transformed into corresponding interval numbers and then the interval objective function has been transformed into a classical multi-objective EPQ (economic production quantity) problem. To minimize the interval objective function, the order relation that represents the decision maker's preference between interval objective functions has been defined by the right limit, left limit, center and half width of an interval. Finally, the transformed problem has been solved by intuitionistic fuzzy programming technique. The proposed method is illustrated with a numerical example and Pareto optimality test has been applied as well.

1. Introduction

Inventory problems are common in manufacturing, maintenance service and business operations in general. Often uncertainties may be associated with demand and various relevant costs like those of carrying, shortage and set-up. In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory, originally introduced by Zadeh (1965), is applicable. Today most of the real-world decision-making problems in economic, technical and environmental ones are multidimensional and multi-objective. It is significant to realize that multiple-objectives are often non-commensurable and are at loggerheads with each other in optimization problem. An objective within exact target value is termed as fuzzy goal. So a multi-objective model with fuzzy objectives is more realistic than deterministic model.

Usually researchers considered different parameters of an inventory model either as constant or as dependent on time or as probabilistic in nature for the development of the EOQ/EPQ model. But, in real life situations, these parameters may have slight deviations from the exact value which may not follow any probability distribution. In these situations, if they are treated as fuzzy parameters, such a model is more realistic. Recently, the concept of fuzzy parameters has been introduced in the inventory problems by several researchers.

In decision making process Bellman and Zadeh (1970), were the first to introduce fuzzy set theory. Tanaka et al. (1974) applied the concept of fuzzy sets to decision-making problems to consider the objectives as fuzzy goals over the α-cuts of fuzzy constraints. Zimmermann (1976, 1978) showed that the classical algorithms can be used in a few inventory models. Li et al. (2002) discussed fuzzy models for single-period inventory problem. Abuo-El-Ata et al. (2003) considered a probabilistic multi-item inventory model with varying order cost. A single-period inventory model with fuzzy demand was analyzed by Kao and Hsu (2002). Fergany and El-Wakeel (2004) considered a probabilistic single-item inventory problem with varying order cost under two linear constraints. A survey of literature on continuously deteriorating inventory models was discussed by Raafat (1991). Hala and El-Saadani (2006) analyzed a constrained single period stochastic uniform inventory model with continuous distributions of demand and varying holding cost. Some inventory problems with fuzzy shortage cost was discussed by Katagiri and Ishii (2000). Moon and Choi (1998) implemented a note on lead time and distributional assumptions in continuous review inventory models. Hariga and Ben-Daya (1999) considered some stochastic inventory models with deterministic variable lead time. A fuzzy EOQ model with demand dependent unit cost under limited storage capacity was implemented by Roy and Maiti (1997). Zheng (1994) discussed optimal control policy for stochastic inventory systems with Markovian discount opportunities. Park (1987), Vujosevic et al. (1996), Chung (2003), Lee and Yao (2004), Lin and Yao (2000), Maiti (2011), Maity and Maity (2006) proposed the EOQ/EPQ

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model in fuzzy sense where inventory parameters are triangular fuzzy numbers (TFNs).

Lai and Hwang (1992, 1994) elaborately discussed fuzzy mathematical programming and fuzzy multiple objective decision making in their two renowned contributions. Ouyang and Chang (2002) analyzed a minimax distribution free procedure for mixed inventory models involving variable lead time with fuzzy lost sales. Teghem et al. (1986) discussed an interactive method for multi-objective linear programming under uncertainty. Mahapatra and Roy (2006) discussed fuzzy multi-objective mathematical programming on reliability optimization model. Deshpande et al. (2011) proposed a bacterial foraging approach to solve fuzzy multi-objective function in inventory management. Ben Abdelaziz (2012) surveyed most solution approaches to the multi-objective stochastic optimization field and pointed out the importance of multi-objective stochastic programming in modeling many practical and complex situations. Based on an order relation of interval number Jiang et al. (2008) suggested a method to solve the nonlinear interval number programming problem with uncertain coefficients both in nonlinear objective function and nonlinear constraints. Steuer (1981), Tang (1994), Ishibuchi and Tanaka (1990), Wolfe (2000) applied mathematical programming inexact, fuzzy and interval programming techniques, to deal with the ambiguous coefficients or parameters in an objective function. The programming technique is more flexible and allows one to find out the solutions which are more or less sufficient for the real problem. In fuzzy optimization the degree of acceptance of objectives and constraints is considered only. Nowadays, the fuzzy set theory has also been developed in a large area and its modifications and generalized forms have appeared. Intuitionistic fuzzy set (IFS) is one of the generalized forms of the fuzzy set. The concept of an IFS can be seen as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of conventional fuzzy sets. Therefore it is expected that IFS could be used to simulate the human decision-making process and any activity requiring human expertise and knowledge which are inevitably imprecise or not totally reliable. Here the degrees of rejection and satisfaction have been considered so that the sum of both values is always less than one.

Atanassov also analyzed intuitionistic fuzzy sets in a more explicit way. Atanassov and Gargov (1989) discussed an open problem in intuitionistic fuzzy sets theory. Nikolova et al. (2002) presented a survey of the research on intuitionistic fuzzy sets. The intuitionistic fuzzy set has received increasingly more attention since its glowing appearance in Wei (2009, 2010) and Wei et al. (2011).

Angelov (1997) implemented the optimization in an intuitionistic fuzzy environment. Angelov (1995) also contributed in another important paper, based on intuitionistic fuzzy optimization. Wei (2008) used the maximizing deviation methods to solve the intuitionistic fuzzy multiple attribute decision making problems with incomplete weight information. Pramanik and Roy (2004) solved a vector optimization problem using an intuitionistic fuzzy goal programming. A transportation model was solved by Jana and Roy (2007) using multi-objective intuitionistic fuzzy linear programming.

In this paper, we propose an inventory model with fuzzy inventory costs and fuzzy demand rate. The said fuzzy parameters are then converted into appropriate interval numbers following Grzegorzewski (2002). We propose a method to solve the EPQ inventory model using the concept of interval. We have constructed an equivalent multi-objective deterministic model corresponding to the original problem with interval coefficients. To obtain the solution of this equivalent problem, we have used intuitionistic fuzzy programming technique where the degree of acceptance (satisfaction) is considered as exponential function and rejection of objectives is considered as quadratic function. Then this intuitionistic fuzzy optimization is converted into a crisp one and the resultant solution becomes a $(\alpha - \beta)$ Pareto optimal solution.

The advantage of the intuitionistic fuzzy optimization technique is twofold: It gives the richest apparatus for formulation of optimization problems and the solutions of intuitionistic fuzzy optimization problems can satisfy the objective(s) in greater degree compared to the analogous fuzzy optimization problem and the crisp one. In order to illustrate the solution method, numerical examples are provided.

2. Preliminaries

Definition 1 (*Atanassov (1986, 1999)*). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An Atanassov's IFS A is a set of ordered triples,

$$A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}$$

where $\mu_A(x_i)$ and $\nu_A(x_i)$ are functions mapping from X into $[0,1]$. For each $x_i \in X, \mu_A(x_i)$ represents the degree of membership and $\nu_A(x_i)$ represents the degree of non-membership of the element x_i to the subset A of X. For the functions $\mu_A(x_i)$ and $\nu_A(x_i)$ mapping into $[0,1]$ the condition $0 \leqslant \mu_A(x_i) + \nu_A(x_i) \leqslant 1$ holds.

Definition 2. Let A and B be two Atanassov's IFSs in the set X. The intersection of A and B is defined as follows:

$$A \cap B = \{\langle x_i, \min(\mu_A(x_i), \mu_B(x_i)), \max(\nu_A(x_i), \nu_B(x_i))\rangle | x_i \in X\}.$$

Definition 3 (*(Interval number): Moore (1979)*). Let $\Re$ be the set of all real numbers. An interval may be expressed as

$$\bar{a} = [a_L, a_R] = \{x : a_L \leqslant x \leqslant a_R, a_L \in \Re, a_R \in \Re\},$$

where a_L and a_R are called the lower and upper limits of the interval $\bar{a}$, respectively. If $a_L = a_R$ then $\bar{a} = [a_L, a_R]$ is reduced to a real number a, where $a = a_L = a_R$. The set of all interval numbers in $\Re$ is denoted by $I(\Re)$.

Basic interval arithmetic: Let $\bar{a} = [a_L, a_R]$ and $\bar{b} = [b_L, b_R] \in I(\Re)$, then

$$\bar{a} + \bar{b} = [a_L + b_L, a_R + b_R].$$

The multiplication of an interval by a real number $c \neq 0$ is defined as

$$c\bar{a} = [ca_L, ca_R];\ \text{if}\ c \geqslant 0\ \text{and}\ c\bar{a} = [ca_R, ca_L];\ \text{if}\ c < 0.$$

The difference of these two interval numbers is

$$\bar{a} - \bar{b} = [a_L - b_R, a_R - b_L].$$

The product of these two distinct interval numbers is given by

$$\bar{a}.\bar{b} = [\min\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\}, \max\{a_L.b_L, a_L.b_R, a_R.b_L, a_R.b_R\}].$$

The division of these two interval numbers with $0 \notin \bar{b}$ is given by

$$\bar{a}/\bar{b} = \left[\min\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\}, \max\left\{\frac{a_L}{b_L}, \frac{a_L}{b_R}, \frac{a_R}{b_L}, \frac{a_R}{b_R}\right\}\right].$$

Optimization in interval environment: Now we defined a general nonlinear objective function with coefficients of the decision variables as interval numbers as

$$\text{Minimize}\ \overline{Z}(x) = \frac{\sum_{i=1}^{n}[a_{L_i}, a_{R_i}]\prod_{j=1}^{k} x_j^{r_j}}{\sum_{i=1}^{l}[b_{L_i}, b_{R_i}]\prod_{j=1}^{n} x_j^{q_j}} \tag{1}$$