European Journal of Operational Research 223 (2012) 60–75

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Multiple pickup and delivery traveling salesman problem with last-in-first-out loading and distance constraints

Brenda Cheang [a], Xiang Gao [b], Andrew Lim [b], Hu Qin [a,*], Wenbin Zhu [c]

[a] School of Management, Huazhong University of Science and Technology, No. 1037, Luoyu Road, Wuhan, China
[b] Department of Management Sciences, City University of Hong Kong, Tat Chee Ave., Kowloon Tong, Kowloon, Hong Kong
[c] Department of Computer Science and Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Received 1 April 2011
Accepted 9 June 2012
Available online 17 June 2012

Keywords:
Traveling salesman
Pickup and delivery
Last-in-first-out loading
Distance constraint
Metaheuristic

ABSTRACT

We extend the traveling salesman problem with pickup and delivery and LIFO loading (TSPPDL) by considering two additional factors, namely the use of multiple vehicles and a limitation on the total distance that a vehicle can travel; both of these factors occur commonly in practice. We call the resultant problem the *multiple pickup and delivery traveling salesman problem with LIFO loading and distance constraints* (MTSPPD-LD). This paper presents a thorough preliminary investigation of the MTSPPD-LD. We propose six new neighborhood operators for the problem that can be used in search heuristics or meta-heuristics. We also devise a two-stage approach for solving the problem, where the first stage focuses on minimizing the number of vehicles required and the second stage minimizes the total travel distance. We consider two possible approaches for the first stage (*simulated annealing* and *ejection pool*) and two for the second stage (*variable neighborhood search* and *probabilistic tabu search*). Our computational results serve as benchmarks for future researchers on the problem.

1. Introduction

The traveling salesman problem with pickup and delivery (TSPPD) is a well-studied problem (e.g., Kalantari et al., 1985; Healy and Moll, 1995; Renaud et al., 2000, 2002; Dumitrescu et al., 2009). In the TSPPD, we are given a set $R = \{1,\ldots,n\}$ of n requests, where each request requires a load to be transported from pickup vertex i^+ to delivery vertex i^-, $i = 1,\ldots,n$. There is a single vehicle with unlimited capacity that starts from a depot vertex 0^+. Its task is to carry out all requests by visiting each pickup vertex before its corresponding delivery vertex (known as the *precedence constraint*) and finally return to a depot vertex 0^- while minimizing the total distance traveled. The TSPPD is defined on a complete weighted undirected graph $G = (V, E)$, where $V = \{0^+,1^+,\ldots,n^+,0^-,1^-,\ldots,n^-\}$ is the vertex set; $E = \{(x, y): x, y \in V, x \neq y\}$ is the edge set; and the edge weight $d(x, y)$ is the non-negative distance between vertices x and y.

A TSPPD variant that has recently received significant attention requires that loading and unloading operations are performed in a last-in-first-out (LIFO) manner, known as the TSPPD with LIFO loading (TSPPDL) (Iori and Martello, 2010). The TSPPDL naturally arises when routing a vehicle whose storage unit has only a single door located at the rear and works like a *stack*; it is especially applicable when the cost of rearranging loaded items is much higher than that of the extra traveling distance caused by the LIFO constraint, i.e., the rearrangement cost dominates the traveling cost. Examples include the transportation of bulky, fragile or hazardous items.

The TSPPDL was first mentioned by Ladany and Mehrez (1984), who solved a real-life delivery problem in Israel using an enumerative approach. Two classes of solution approaches have been applied to the TSPPDL, namely exact algorithms (Carrabs et al., 2007a; Cordeau et al., 2010) and heuristics (Pacheco, 1997; Levitin and Abezgaouz, 2003; Cassani and Righini, 2004; Carrabs et al., 2007b; Li et al., 2011). The difficulty of the TSPPDL is evidenced by the fact that the largest instances optimally solved in the literature contain only 25 requests (Cordeau et al., 2010), so heuristic approaches are required to handle practical instances with hundreds of requests. Currently, the best and latest heuristic is the variable neighborhood search heuristic proposed by Li et al. (2011) that employs rooted ordered trees to represent the feasible solutions of the TSPPDL.

A direct generalization of the TSPPDL is the single vehicle pickup and delivery problem with multiple stacks (1-PDPMS) that considers a vehicle with multiple independent stacks (Côté et al., 2009).

* Corresponding author. Tel.: +852 64117909/+86 13349921096; fax: +86 27 87556437.

E-mail addresses: brendacheang@yahoo.com (B. Cheang), gaoxiang@cityu.edu.hk (X. Gao), lim.andrew@cityu.edu.hk (A. Lim), tigerqin@mail.hust.edu.cn, tigerqin1980@gmail.com (H. Qin), i@zhuwb.com (W. Zhu).

0377-2217/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2012.06.019

Moreover, a special case of the 1-PDPMS which specifies all pickups to be performed before all deliveries was recently proposed by Petersen and Madsen (2009); this problem is called the double traveling salesman problem with multiple stacks (DTSPMS), which has also been solved by exact algorithms (Lusby et al., 2010; Petersen et al., 2010; Alba et al., in press; Carrabs et al., in press) and heuristics (Felipe et al., 2009a,b, 2011; Petersen and Madsen, 2009).

This paper introduces another useful and practical generalization of the TSPPDL by considering two additional factors. The first is to allow multiple vehicles, which reflects the fact that practical problems of this nature usually involve a fleet of vehicles rather than a single vehicle. The second is to require that the route length of each vehicle cannot exceed a predetermined limit L, which stems from regulations on working hours for drivers. For example, European Union regulation (EC) No. 561/2006 stipulates that the daily driving time shall not exceed 9 h, while in the United States the Federal Motor Carrier Safety Administration (FMCSA) mandates that commercial motor vehicle drivers may only drive up to 11 cumulative hours in a 14-h period. The distance a driver can travel is also limited if they have to return their vehicle to the depot by the end of the working day. This type of distance constraint has been widely applied to many vehicle routing problem (VRP) variants (see Laporte et al., 1985; Li et al., 1992; Nagy and Salhi, 2005; Erera et al., 2010).

We call the resultant problem the *multiple pickup and delivery traveling salesman problem with LIFO loading and distance constraints* (MTSPPD-LD). A solution to this problem is a set of vehicle routes that satisfy the precedence, LIFO and distance constraints. The primary objective of the MTSPPD-LD is to minimize the number of vehicle routes required to fulfill all requests. Once this is achieved, the secondary objective is to minimize the total distance traveled.

In this paper, we use the tree representation of feasible vehicle routes suggested by Li et al. (2011) to design several new search operators for this problem, some of which were adapted from the traditional move operators for the VRP with time windows (VRPTW), such as *relocate, exchange* and *CROSS-exchange* (Bräysy and Gendreau, 2005). These operators are employed in a two-stage heuristic: the first stage concentrates on reducing the number of trees (i.e., vehicle routes) from an initial solution, and the second stage focuses on minimizing the total distance traveled. This strategy of separating vehicle reduction from distance minimization has been successfully employed on other transportation problems (Bent and Van Hentenryck, 2004, 2006; Homberger and Gehring, 2005; Lim and Zhang, 2007).

For the first stage, we designed two vehicle reduction algorithms; one is a simulated annealing algorithm and the other is based on the concept of an ejection pool (Lim and Zhang, 2007; Nagata and Bräysy, 2009). We also designed two distance minimization algorithms for the second stage, namely a variable neighborhood search (VNS) heuristic (Mladenović and Hansen, 1997) and a probabilistic tabu search heuristic (Erdoğan et al., 2009). All of these approaches have been previously successfully applied to related problems. By implementing and examining the effectiveness of these techniques, our aim is to provide a foundation upon which future researchers of this problem can build. To test our approaches, we generated a large number of test instances based on the TSPPDL instances introduced by Carrabs et al. (2007b). Our experiments on these test instances suggest that the best two-stage heuristic consists of using the ejection pool algorithm for vehicle reduction and VNS for distance reduction.

To the best of our knowledge, the only existing research on the MTSPPD-LD is presented in Gao et al. (2011), which designed a VNS heuristic to minimize the total distance for the MTSPPD-LD with a fixed number of free vehicles (i.e., no fixed charge for the used vehicle). Gao et al. (2011) observed that if two trees can be merged without violating the distance constraint, the total distance must be reduced. Therefore, they incorporated a *tree-merge* operator in their VNS heuristic. We do not adopt this operator in the second stage of our approach because the possibility of successfully merging two trees is negligible after the number of trees is minimized in the first stage. Moreover, Gao et al. (2011) used a dynamic programming algorithm to optimize the individual tree with 13 or fewer requests. However, we found by some preliminary experiments that the VNS heuristic proposed in Li et al. (2011) (henceforth referred to as the VNS-SingleTree heuristic) can achieve optimal solutions for almost all TSPPDL instances with such scale using much less computation times; thus, we only apply VNS-SingleTree heuristic when optimizing individual trees.

The remainder of this paper is organized as follows. In Section 2, we briefly explain the tree representation of a feasible vehicle route that we use in our approach (originally proposed by Li et al. (2011)). We then introduce six new inter-tree operators in Section 3; these operators are employed by the various heuristics that we examine in this study. Section 4 describes two possible algorithms for the vehicle reduction stage of our approach, namely a simulated annealing algorithm and an ejection pool algorithm. Similarly, Section 5 describes two possible algorithms for the distance reduction stage, namely a VNS algorithm and a probabilistic tabu search. The effectiveness of these algorithms is analyzed with computational experiments that are detailed in Section 6, where we also explain how our test instances were generated and how the various parameters of the component algorithms were determined. We conclude our study in Section 7 with some additional observations and possible future research directions.

2. The tree representation of feasible routes

A feasible solution of the MTSPPD-LD is a set of vehicle routes, which can be simply represented by a set of vertex sequences; Fig. 1 shows a feasible solution for an MTSPPD-LD instance comprising two routes. Most heuristics for vehicle routing and scheduling problems are based on edge-exchange operations (Toth and Vigo, 2002), which involve the moving or exchanging of edges in vertex sequences. However, if such operations are directly applied to our problem, the resultant routes usually violate the precedence or LIFO constraint (or both). While it is possible to design sequence-based search operators for the problem, considerable effort has to be made to guarantee the feasibility of the resultant routes; this is clearly demonstrated by the complex additional checks required for the sequence-based VNS approach for the TSPPDL by Carrabs et al. (2007b).

To resolve this issue, we represent a feasible solution of the MTSPPD-LD by a *forest*, which consists of a set of ordered trees. Each tree represents a vehicle route with length less than L, and

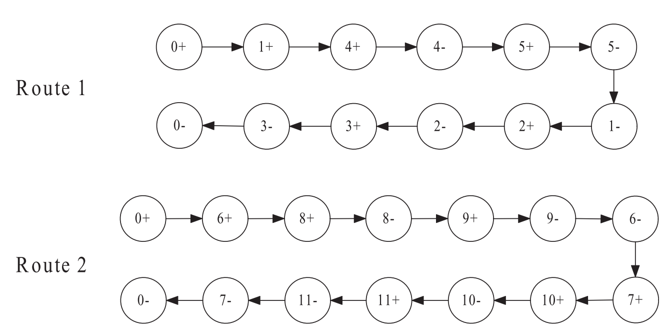

Fig. 1. The sequence representation of a feasible solution.