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Decision Support

A piecewise linearization framework for retail shelf space management models

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ABSTRACT

Managing shelf space is critical for retailers to attract customers and optimize profits. This article develops a shelf-space allocation optimization model that explicitly incorporates essential in-store costs and considers space- and cross-elasticities. A piecewise linearization technique is used to approximate the complicated nonlinear space-allocation model. The approximation reformulates the non-convex optimization problem into a linear mixed integer programming (MIP) problem. The MIP solution not only generates near-optimal solutions for large scale optimization problems, but also provides an error bound to evaluate the solution quality. Consequently, the proposed approach can solve single category-shelf space management problems with as many products as are typically encountered in practice and with more complicated cost and profit structures than currently possible by existing methods. Numerical experiments show the competitive accuracy of the proposed method compared with the mixed integer nonlinear programming shelf-space model. Several extensions of the main model are discussed to illustrate the flexibility of the proposed methodology.

1. Introduction

The choice of which products to stock among numerous competing products and how much shelf space to allocate to those products are central decisions for retailers. Because shelf space is a scarce and fixed resource and potentially available products are continually increasing, retailers have a high incentive to make these decisions using optimization tools. Brand-loyal customers look for a specific item and buy it if it is available or delay if is not. Thus, space allocated to a product has no effect on its sales (Anderson, 1979). However, marketing research shows that most customer decisions are made at the point of purchase (POPAI, 1997). Ehrenberg (1972) discovered that, "except in relatively short time periods ... buyers of any particular brand therefore buy other brands more often than the brand itself." This indicates that in-store factors, including shelf space allocated to a product, may influence customer product choice. Retailers with a well-designed shelf space management system can, therefore, attract customers, prevent stock outs, and increase store financial performance while reducing operating costs (Yang and Chen, 1999). In addition, close-to-optimal shelf space allocations provide the basis for distributing promotional resources among different product categories (Chen et al., 1999). However, because products typically have different profit margins and vary widely in space- and cross-elasticities, the optimization problem is very complicated to solve.

This article presents realistic shelf-space management optimization models, provides a solution procedure that can handle practical problem sizes, and is flexible enough to be applied to a wide range of shelf-space management models. The current work builds on the well-known model of Corstjens and Doyle (1981) and extends it in three directions. First, the proposed model allocates shelf-space to a product as an integer number of its facing, which is what store customers see from the product. Second, the model allows for simultaneous shelf space and assortment decisions. Third, cost elements, such as assortment and replenishment costs, are modeled for individual products. The proposed solution methodology reformulates the nonconvex mixed integer nonlinear programming (MINLP) model using piecewise linear functions. Thus, the reformulated model can be solved easily by the commonly used linear mixed integer programming (MIP) technique. The reformulated model generates both a feasible solution and a bound on the (globally) optimal objective value of the exact nonlinear model. This allows for the calculation of an *a posteriori* error bound[1] on the optimal MIP solution. This article also extends the model to incorporate the following additional effects mentioned in the literature: marketing variables other than space (Yang and Chen, 1999), fixed procurement costs and the possibility of storing items in

[1] Some heuristics (none proposed in the literature for shelf space management models) guarantee a *priori* worst case bounds on solutions to any instance of a problem. A *posteriori* bounds are problem-specific and can only be evaluated after a problem is solved.

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a warehouse (Urban, 1998), and substitution effects due to temporary or permanent unavailability of products (Borin et al., 1994).

The remainder of this article is organized as follows: Section 2 reviews the relevant literature and develops the main shelf space management model. Section 3 develops the piecewise linearization model that reformulates the complex MINLP model. Section 4 presents real-life data test results, sensitivity analyses, and computational experiments for a large scale problem. Section 5 discusses various extensions of the main model and their linearizations.

This article demonstrates that the proposed linearization technique can be applied to a wide range of shelf-space management models in which the demand function is of signomial form. To clarify the presentation, standardized notations are used in this paper as follows: decision variables and variables that depend on decision variables are written in lower-case Roman characters; constants and given quantities are indicated by upper-case Roman characters; parameters and quantities that need to be estimated or user-provided are expressed in lower-case Greek characters; and functions, without arguments, are specified by upper-case Greek characters.

2. Literature review

The discussion below focuses on literature that addresses *models* and *procedures* related to shelf-space allocation problems considered in this article.

2.1. Commercial models

In the retail industry, commercial software incorporating various cost models is popular due to its general simplicity and easily implementable decisions (Zufryden, 1986). Examples include Apollo (IRI) and Spaceman (Nielsen), which are PC based programs. These software systems provide retailers a realistic view of the shelves and are capable of allocating shelf space according to simple heuristics, such as turnover, gross profit or margin, and constraints from handling and inventory costs (Desmet and Renaudin, 1998). The main drawbacks of all these systems is their failure to include demand effects, that is, the systems ignore the effects of shelf space allocation on product sales. Thus, as addressed in Desmet and Renaudin (1998), none of these systems can be considered as serious tools for optimizing shelf space allocations. Consequently, it is not surprising that most retailers "use them mainly for planogram accounting purposes to reduce the amount of time spent on manually manipulating the shelves" (Drèze et al., 1994).

2.2. Optimization models

Hansen and Heinsbroek (1979) developed one of the first shelf-space allocation optimization models using a multiplicative demand function that incorporates individual space-elasticities, but disregards cross-elasticities from similar products. Constraints of total available shelf space, minimum allocations, and integer solutions are considered. Binary variables for handling assortment decisions are also included. The authors applied a generalized Lagrange multiplier to solve the optimization problem; however, it is only guaranteed to find local solutions for non-convex programs.

The model of Corstjens and Doyle (1981) incorporates both space- and cross-elasticities and accounts for constraints similar to those considered by Hansen and Heinsbroek (1979). However, the Corstjens and Doyle model incorporates a more detailed cost structure including procurement costs, carrying costs, and out-of-stock costs, which are jointly modeled as a multiplicative form to allocated shelf space. A signomial geometric programming approach is used to optimize the shelf space allocation; however, Borin et al. (1994) indicated that the reported solutions for seven of ten problems violated the model constraints.

The SHARP model developed by Bultez and Naert (1988) and Bultez et al. (1989) is similar to the one developed by Corstjens and Doyle (1981). However, they did not develop an explicit function relating shelf space allocation to product sales. Instead, space elasticities are estimated using a symmetric attraction model for SHARP-1 and an asymmetric model for SHARP-2. A heuristic procedure is proposed to optimize these models.

Borin et al. (1994) extended the demand function of Corstjens and Doyle (1981) to allow simultaneous decisions about assortment selections and shelf space allocations. They explicitly considered substitution effects due to temporary or permanent unavailability of products. The resulting model optimizes return on inventory and is solved using simulated annealing heuristic procedures. Borin and Farris (1995) analyzed the degree of errors that may be introduced in estimating parameters required for Borin et al. (1994).

Yang and Chen (1999) simplified the Corstjens and Doyle model (1981), disregarding cross-elasticities and assuming that a product's profit is linear within a small number of facings. They allowed the profit of each product to vary when allocated to different shelves by formulating the shelf-space allocation problem, similar to a knapsack problem. Allowing profit to depend on shelf placement is consistent with the experimental study of Drèze et al. (1994), who concluded that product location on shelves is more important for determining product sales than the amount of space allocated to the product. Yang (2001) proposed a heuristic to optimize this model. His technique extends an approach applied to solve simple knapsack problems. Lim et al. (2004) combined a local search technique with meta-heuristics to optimize the Yang and Chen model (1999), extending the model to account for profit functions and product groupings.

Although the research directions of Borin et al. (1994) and Yang and Chen (1999) fundamentally differ, both share the following weaknesses. They focus on the revenue side and do not explicitly incorporate the cost side of operations. Clearly, certain costs are not independent of shelf space allocation. For example, the smaller the shelf space allocated to a product, the greater the restocking frequency and the higher the resulting restocking costs for the product. Urban (1998) generalized the inventory-dependent demand model to explicitly model the demand rate as a nonlinear function of the inventory level. He used a greedy heuristic and a genetic algorithm to solve the product assortment and shelf-space allocation. Hwang et al. (2005) formulated a nonlinear programming model for a replenishment problem when item demand rate is a function of quantity and display location on the shelves. A gradient search heuristic and a genetic algorithm were also proposed to solve the problem. Note that heuristics are used to solve many of these optimization problems. Although the nonlinear programming model might provide good feasible solutions for the test cases considered, it cannot guarantee an optimal or close-to optimal solution. All heuristics-based solutions do not provide a method to determine how good the computed solution actually is to the globally optimized solution.

Several other articles have been published in a few related areas. For example, Kok and Fisher (2004) developed an assortment-planning model where consumers may accept substitutes when their favorite product is unavailable. They presented a methodology for estimating model parameters using sales data from