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Decision Support

ELECTRE^{GKMS}: Robust ordinal regression for outranking methods

Salvatore Greco[a], Miłosz Kadziński[b,*], Vincent Mousseau[c], Roman Słowiński[b,d]

[a] Faculty of Economics, University of Catania, Corso Italia, 55, 95129 Catania, Italy
[b] Institute of Computing Science, Poznań University of Technology, 60-965 Poznań, Poland
[c] Laboratoire Génie Industriel, Ecole Centrale Paris, Grande Voie des Vignes, 92 295 Châtenay-Malabry Cedex, France
[d] Systems Research Institute, Polish Academy of Sciences, 01-447 Warsaw, Poland

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ABSTRACT

We present a new method, called ELECTRE^{GKMS}, which employs robust ordinal regression to construct a set of outranking models compatible with preference information. The preference information supplied by the decision maker (DM) is composed of pairwise comparisons stating the truth or falsity of the outranking relation for some real or fictitious reference alternatives. Moreover, the DM specifies some ranges of variation of comparison thresholds on considered pseudo-criteria. Using robust ordinal regression, the method builds a set of values of concordance indices, concordance thresholds, indifference, preference, and veto thresholds, for which all specified pairwise comparisons can be restored. Such sets are called compatible outranking models. Using these models, two outranking relations are defined, necessary and possible. Whether for an ordered pair of alternatives there is necessary or possible outranking depends on the truth of outranking relation for all or at least one compatible model, respectively. Distinguishing the most certain recommendation worked out by the necessary outranking, and a possible recommendation worked out by the possible outranking, ELECTRE^{GKMS} answers questions of robustness concern. The method is intended to be used interactively with incremental specification of pairwise comparisons, possibly with decreasing confidence levels. In this way, the necessary and possible outranking relations can be, respectively, enriched or impoverished with the growth of the number of pairwise comparisons. Furthermore, the method is able to identify troublesome pieces of preference information which are responsible for incompatibility. The necessary and possible outranking relations are to be exploited as usual outranking relations to work out recommendation in choice or ranking problems. The introduced approach is illustrated by a didactic example showing how ELECTRE^{GKMS} can support real-world decision problems.

1. Introduction

Outranking relation, usually denoted by S, was proposed by Roy whose aim was to establish a realistic representation of four basic situations of preference: indifference, weak preference, strict preference, and incomparability [18]. Roy also claimed that a satisfactory preference model could be developed by assigning one, two or three of these four basic situations to any pair of alternatives. Thus, stating that a outranks b, which is equivalent to saying that a is at least as good as b, the DM does not have to decide whether this means that a is indifferent to, weakly preferred to, or strictly preferred to b. The assertion aSb is considered to be true if there are sufficient arguments to affirm that a is not worse than b, and if there is no essential reason to refuse this assertion. The first condition is called *concordance test*, and the second, *non-discordance test*. On the other hand, the outranking relation for a pair of alternatives (a,b) is false, and denoted by aS^Cb, either if the concordance test or the non-discordance test is negative.

The usefulness of the outranking model comes from the fact that it is based on relatively weak mathematical assumptions [21]. It attempts to enrich the dominance relation by strongly established preferences, accepting incomparability and neither imposing completeness nor transitivity. On the other hand, results following its usage are less conclusive than outcomes of multi-attribute utility (or value) theory. However, the latter one is often referred to as not really reliable due to violation of too strong mathematical assumptions in practical applications [22].

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* Corresponding author. Tel.: +48 608279303.

E-mail addresses: salgreco@unict.it (S. Greco), milosz.kadzinski@cs.put.poznan.pl (M. Kadziński), vincent.mousseau@ecp.fr (V. Mousseau), roman.slowinski@cs.put.poznan.pl (R. Słowiński).

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The preference model in the form of an outranking relation is most widely used in the ELECTRE family of Multiple Criteria Decision Aiding (MCDA) methods. All those methods are based on the same rule: knowing S and S^C for all ordered pairs of alternatives, one proceeds to exploitation of the outranking relation, which is specific for choice or ranking problem. According to Figueira et al. [8] ELECTRE methods are relevant when facing decision situations where the DM wants to consider at least three, but preferably more than five and less than a dozen or so criteria. Additionally, ELECTRE methods are well suited when at least one of the following four conditions is satisfied. First of all, at least one criterion is coded in an ordinal scale or an ordered metric scale [4], which is also called a weak interval scale [13]. Further, a strong heterogeneity among the nature of evaluations on different criteria exists or trade-offs between criteria may not be acceptable for the DM. Finally, for at least one criterion only the accumulation of several small differences of evaluations may become significant in terms of preferences, while a single small difference is not significant.

There exist various outranking methods which differ in the way of formalizing the general concept of concordance and non-discordance as well as in the way of exploiting the outranking relation in different decision problems. Two major problems concerning outranking methods have been raised in the literature. The first one concerns elicitation of preference information necessary for construction of the outranking relation. It is not an easy task for a DM, because it requires fixing a precise numerical value for such parameters as importance coefficients (weights) of criteria, indifference, preference, and veto thresholds. Moreover, some technical parameters need to be fixed in advance, like concordance threshold, distillation parameters, etc. For this reason, some disaggregation-aggregation procedures have been proposed to assist the elicitation of the weights of criteria and all the thresholds required to build the model (e.g., [16,17]). The other problem refers to a robustness analysis. For instance, it has been suggested that assignment of a non-negligible amplitude of numerical values to the thresholds and importance coefficients could be followed by consideration of central values to obtain a first solution [19,22]. Then, one should take advantage of different combinations of the extreme values, which would allow examination of the influence of each parameter on the final outcomes, and indication of the solutions which are good for different sets of parameter values.

Previous advances in robustness analysis for ELECTRE methods have been presented by Dias and Clímaco [5,6]. They consider the case when only partial information on the parameter values is provided. Precisely, if the DM has difficulties in fixing precise values for all parameters required by ELECTRE, they allow her/him to provide constraints on the acceptable combinations of these parameter values. Subsequently, they investigate whether an outranking among two alternatives is robust. They define relations denoted by S^R and N^R, which represent outranking or lack of outranking for all admissible combinations of parameter values, respectively. They also propose the array of tools for enriching such robust conclusions in case they are too poor, and for exploiting them to work out the final recommendation. In this paper, we reconsider those two appealing issues, i.e. elicitation of preference information and analysis of robustness, and discuss a new general framework for disaggregation–aggregation approach to outranking methods, called ELECTRE^{GKMS}.

ELECTRE^{GKMS} can be seen as an inherent part of the *robust ordinal regression* paradigm, which has been recently developed with the aim of taking into account all instances of a preference model compatible with the preference information and, consequently, supplying the decision maker with two kinds of results, necessary and possible [12]. The first method based on this paradigm was UTA^{GMS} [10], further generalized in another approach called GRIP [7]. Both those methods are designed to support multiple criteria ranking problems. Robust ordinal regression has been also applied to Choquet integral with the aim of representing interactions between criteria [1]. Further, UTA-DIS^{GMS} was developed by analogy to UTA^{GMS} to deal with multiple criteria sorting of alternatives [11]. The aforementioned UTA-like methods apply the idea of robust ordinal regression to general additive value functions. The aim of this paper is to show that this principle is independent of the type of preference model involved, and to generalize outranking methods in a few aspects:

- taking into account *all outranking models compatible with the provided preference information*, and not only one such relation,
- considering the marginal concordance functions as *general monotonic* ones, defined in the "spirit" of ELECTRE methods with clearly distinguished areas of strict and weak preference as well as indifference, and not only piecewise linear or of other arbitrarily chosen shape,
- handling of preference information composed of *pairwise comparisons stating the truth or falsity of a small set of reference alternatives*, and of *imprecise intra-criterion preference information* which can be supplied in a direct or indirect way,
- constructing four relations in the set of alternatives A, such that for any pair of alternatives $(a,b) \in A \times A$: *a necessarily* outranks b (aS^Nb) if and only if aSb for *all* outranking models compatible with the preference information, *a possibly* outranks b (aS^Pb) if and only if aSb for *at least one* compatible outranking model, *a necessarily* does not outrank b ($aS^{CN}b$) if and only if aS^Cb for *all* compatible models, and *a possibly* does not outrank b ($aS^{CP}b$) if and only if aS^Cb for *at least one* compatible model.

Thus defined, the necessary relations specify the most certain recommendation worked out on the basis of all compatible outranking models, while the possible relations identify a recommendation provided by at least one compatible outranking model. Consequently, the necessary outcomes can be considered as robust with respect to the preference information, as they guarantee that a definite relation is the same whichever compatible outranking model would be used.

The form of required preference information and the nature of provided results gives the space for interactivity with the DM. The necessary and possible outranking relations can be, respectively, enriched or impoverished with the growth of the number of pairwise comparisons. Notice that in this paper, we rather focus on the first stage out of the traditionally distinguished two steps of outranking methods, i.e. building of the outranking relation. We suggest exploitation of the necessary and possible relations with regard to the chosen statement of the problem by means of exploitation procedures used in existing ELECTRE-like methods.

The organization of the paper is the following. In the next section, we introduce notation that will be used along the paper. The principle of robust ordinal regression applied to outranking methods is outlined in Section 3. It consists of the definition of the model used in ELECTRE^{GKMS}, the procedures for verification of the truth of necessary and possible outranking relations, and discussion of some of their properties. In Section 4, we consider some extensions of the method including analysis of incompatibility, specification of pairwise comparisons with gradual confidence levels, discussion of other types of preference information that could be easily incorporated into the framework, and description of some basic procedures for exploitation of the necessary and possible results. Section 5 provides illustrative examples showing how the presented method can be applied in practice. The last section concludes the paper, outlining also some possible ways of future development of the presented method.