



Invited Review

Cooperative game theory and inventory management[☆]M.G. Fiestras-Janeiro^a, I. García-Jurado^{b,*}, A. Meca^c, M.A. Mosquera^d^a Department of Statistics and Operations Research, Faculty of Economics, Vigo University, 36271 Vigo, Spain^b Department of Mathematics, Faculty of Computer Science, Coruña University, 15071 Coruña, Spain^c Operations Research Center, Miguel Hernández University, 03202 Elche, Alicante, Spain^d Department of Statistics and Operations Research, Faculty of Business Administration and Tourism, Vigo University, 32004 Ourense, Spain

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ABSTRACT

Supply chain management is related to the coordination of materials, products and information flows among suppliers, manufacturers, distributors, retailers and customers involved in producing and delivering a final product or service. In this setting the centralization of inventory management and coordination of actions, to further reduce costs and improve customer service level, is a relevant issue. In this paper, we provide a review of the applications of cooperative game theory in the management of centralized inventory systems. Besides, we introduce and study a new model of centralized inventory: a multi-client distribution network.

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1. Introduction

Game theory is the mathematical theory of interactive decision situations. In one of those situations some agents make decisions, depending on their decisions an outcome results, and each agent has his own preferences on the set of possible outcomes. Since one important class of interactive decision situations are parlour games, game theory uses their terminology to designate the elements of the interactive decision situations: these situations are called games, the agents are called players, the agents' plans to make decisions are called strategies, etc. This was already done by John von Neumann and Oskar Morgenstern in their pioneering book "The Theory of Games and Economic Behavior".

Sometimes we are interested in the strategic analysis of games. In that case, we need to model carefully all the relevant aspects of the problem and then look for the best strategies of each player taking into account that the others will also behave searching for their best. We say then that we are adopting a non-cooperative view and should use an appropriate non-cooperative model to perform our analysis. In other cases we just want to deal with the cooperation issues of the problem at hand and propose how the

agents must allocate the benefits of their cooperation. This approach assumes that the agents have mechanisms to enforce their cooperation: it is the cooperative approach.

Operational research models are mathematical instruments to solve decision problems. Most of them deal with one decision maker situations. However, in real world, it is very common that the result of our decisions depend also on other decision makers' choices, i.e. in the real world many decision situations are interactive. Thus, one challenging field within operations research is that of game theoretical models in operations research.

In particular, operations management focused on single-firm analysis in the past. Its goal was to provide managers with suitable tools to improve the performance of their firms. Nowadays, business decisions are dominated by the globalization of markets and should take into account the increasing competition among firms. Further, more and more products reach the customer through supply chains that are composed of independent firms. Following these trends, research in supply chain has shifted its focus from single-firm analysis to multi-firm analysis, in particular to improving the efficiency and performance of supply chains under decentralized control. The main characteristics of such chains are that the firms in the chain are independent actors who try to optimize their individual objectives, and that the decisions taken by a firm do also affect the performance of the other parties in the supply chain. These interactions among firms' decisions ask for alignment and coordination of actions and, therefore, game theory is very well suited to deal with these interactions. This has been recognized by researchers in the field, since there is an ever increasing number of papers that apply tools, methods and models from game

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theory to supply chain problems. A tutorial on the subject is Cachon and Netessine (2004). The authors discuss both non-cooperative and cooperative game theory in static and dynamic settings. Additionally, Cachon (1998) reviews competitive supply chain inventory management, and Cachon (2003) reviews and extends the supply chain literature on the management of incentive conflicts with contracts. Papers using cooperative game theory to study supply chain management are scarce, but the use of cooperative games in this context is becoming more popular. Nagarajan and Sošić (2008) review and extend the problem of bargaining and negotiations in supply chain relationships. A very recent survey on applications of cooperative game theory to supply chain management, the so called supply chain collaboration, is Meca and Timmer (2008). For theoretical issues and a framework for more general supply chain networks we refer to the book by Slikker and van den Nouweland (2001).

An important aspect of supply chain management is a good management of the inventories by the firms or retailers. The management of inventory, or inventory management, started at the beginning of 20th century when manufacturing industries and engineering grew rapidly. As far as we know, a starting paper on mathematical models of inventory management was Harris (1913). Since then, many books on this subject have been published (i.e. Hadley and Whitin, 1963; Hax and Candea, 1984; Tersine, 1994; Zipkin, 2000). Most often, the objective of inventory management is to minimize the average cost per time unit (in the long run) incurred by the inventory system, while guaranteeing a pre-specified minimal level of service.

In the last years, several papers dealing with the applications of cooperative game theory in inventory management have appeared. Though this is a young field, there are already some relevant contributions. In this paper, we review this literature.¹ Besides, we introduce and study a new model to analyse the cooperation in a multi-client distribution network.

2. Cooperation in deterministic inventory situations

This section tackles the study of cooperation in deterministic inventory situations. There are several papers studying various models of cooperation in this area. To start with, we analyse one of the simplest ones, in which all the agents involved in the inventory situation agree to cooperate and the characteristic function is given by an explicit formula. Later on, we review some models of cooperation where the characteristic function is given by the optimal value of an optimization problem. Finally, we deal with situations where the cooperation among the agents is not an assumption and the main issue is to analyse the coalition formation process.

2.1. Characteristic function given by an explicit formula

When several agents face similar inventory problems they may make some savings if they cooperate. For instance, if there is a fixed cost per order, agents will pay less if they order simultaneously as a group than if they make their orders separately. This raises an allocation problem: how should these savings be divided among the agents? This problem was analysed in Meca et al. (2003), on which part of this subsection is based.

¹ When making the first revision of this paper we found another (unpublished) survey of inventory games by Dror and Hartman (2008). Both surveys are very different and concentrate on different problems and classes of games. Our paper, moreover, includes Section 4 which analyses a completely new model.

Assume that there are n agents, $N = \{1, \dots, n\}$, each of them facing an Economic Production Quantity (EPQ) problem with shortages.² An EPQ model with shortages considers an agent i who places orders of a certain good that he sells. The (deterministic) demand that he must fulfill equals to d_i units per time unit ($d_i \geq 0$). The cost of keeping in stock one unit of this good per time unit is h_i ($h_i > 0$). The fixed cost of an order is a . Agent i considers the possibility of not fulfilling all the demand in time, but allowing a shortage of the good. The cost of a shortage of one unit of the good for one time unit is $s_i > 0$. When an order is placed, after a deterministic and constant lead time (which can be assumed to be zero, without loss of generality), agent i receives the order gradually; more precisely, r_i units of the good are received per time unit. It is assumed that $r_i > d_i$ (otherwise the model makes little sense). We call r_i the replacement rate of agent i . The agent must choose an order size \hat{Q}_i and a maximum shortage \hat{M}_i minimizing his average inventory cost per time unit given by:

$$C(Q_i, M_i) = a \frac{d_i}{Q_i} + h_i \frac{(Q_i(1 - \frac{d_i}{r_i}) - M_i)^2}{2Q_i(1 - \frac{d_i}{r_i})} + s_i \frac{M_i^2}{2Q_i(1 - \frac{d_i}{r_i})}.$$

By using elementary mathematical techniques it results that:

$$\hat{Q}_i = \sqrt{\frac{2ad_i}{h_i(1 - \frac{d_i}{r_i})} \frac{h_i + s_i}{s_i}} \quad \text{and} \quad \hat{M}_i = \sqrt{\frac{2ad_i h_i}{s_i(h_i + s_i)} \left(1 - \frac{d_i}{r_i}\right)}.$$

Moreover, by denoting $\hat{m}_i = \frac{d_i}{\hat{Q}_i}$ as the optimal number of orders that i must place per time unit,

$$C(\hat{Q}_i, \hat{M}_i) = 2a\hat{m}_i.$$

Now, assume that the agents in a coalition $S \subset N$ decide to place their orders jointly to save part of the ordering costs; so they spend a instead of $|S|a$ every time an order is placed. We claim that, in order to minimize the sum of the average inventory costs per time unit, the agents must coordinate their orders, so $\frac{Q_i^*}{d_i} = \frac{Q_j^*}{d_j}$ for all $i, j \in N$, Q_i^* and Q_j^* denoting the optimal order sizes for i and j if agents in S cooperate. To see this suppose that, optimally, firm 1 has a longer cycle than firm 2. Then, overall costs decrease when firm 1 shortens its cycle length to that of firm 2. Indeed, the overall ordering cost decreases because few orders are placed and holding costs decreases, because the level of the inventory of firm 1 goes down.

Then, the total average cost per time unit can be written as follows,

$$C(Q_1, (M_j)_{j \in S}) = \frac{ad_1}{Q_1} + \frac{1}{2} \sum_{j \in S} h_j \left(\frac{d_j}{d_1} Q_1 \left(1 - \frac{d_j}{r_j}\right) - 2M_j \right) + \frac{1}{2} \sum_{j \in S} \left((h_j + s_j) \frac{d_1 M_j^2}{d_j Q_1 \left(1 - \frac{d_j}{r_j}\right)} \right).$$

Using standard techniques of differential analysis, it can be checked that the optimal values which minimize C are given by:

$$Q_i^* = \sqrt{\frac{2ad_i^2}{\sum_{j \in S} d_j h_j \frac{s_j}{h_j + s_j} \left(1 - \frac{d_j}{r_j}\right)}} \quad \text{and} \quad M_i^* = Q_i^* \frac{h_i \left(1 - \frac{d_i}{r_i}\right)}{h_i + s_i}$$

for all $i \in S$. Moreover,

$$C(Q_1^*, (M_j^*)_{j \in S}) = 2a \sqrt{\sum_{j \in S} \hat{m}_j^2}.$$

² An exhaustive analysis of the EPQ model with shortages can be found in Tersine (1994).

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