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A hybrid evolutionary algorithm for the periodic location-routing problem

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ABSTRACT

The well-known vehicle routing problem (VRP) has been studied in depth over the last decades. Nowadays, generalizations of VRP have been developed for tactical or strategic decision levels of companies but not both. The tactical extension or periodic VRP (PVRP) plans a set of trips over a multiperiod horizon, subject to frequency constraints. The strategic extension is motivated by interdependent depot location and routing decisions in most distribution systems. Low-quality solutions are obtained if depots are located first, regardless of the future routes. In the location-routing problem (LRP), location and routing decisions are tackled simultaneously. Here for the first time, except for some conference papers, the goal is to combine the PVRP and LRP into an even more realistic problem covering all decision levels: the periodic LRP or PLRP. A hybrid evolutionary algorithm is proposed to solve large size instances of the PLRP. First, an individual representing an assignment of customers to combinations of visit days is randomly generated. The evolution operates through an Evolutionary Local Search (ELS) on visit day assignments. The algorithm is hybridized with a heuristic based on the Randomized Extended Clarke and Wright Algorithm (RECWA) to create feasible solutions and stops when a given number of iterations is reached. The method is evaluated over three sets of instances, and solutions are compared to the literature on particular cases such as one-day horizon (LRP) or one depot (PVRP). This metaheuristic outperforms the previous methods for the PLRP.

1. Introduction

In the near future, companies will be called onto pay a great attention to their transport policy. Indeed, they will have to plan activities involving strategic, tactical and operational decisions under the new challenges of sustainable development. Among these decisions, depot location and vehicle routing are crucial choices. They are usually tackled separately to reduce the complexity of the overall problem. However, research has shown that this strategy often leads to suboptimal solutions (Salhi and Rand, 1989). The location-routing problem (LRP) integrates these two decision levels. In general, the LRP is formulated as a deterministic node routing problem (i.e., customers are located on nodes of the network) and as shown in Min et al. (1998), most of the published papers consider either capacitated routes or capacitated depots (Albareda-Sambola et al., 2005; Tuzun and Burke, 1999). However, some authors have studied stochastic cases (Laporte et al., 1989; Chan et al., 2001; Barreto, 2004) and arc routing versions (Ghiani and Laporte, 2001; Labadi, 2003), and, very recently, a few of them have begun to handle the general LRP with capacities on both depots and routes (Wu et al., 2002; Barreto, 2004; Prins and Prodhon, 2004a; Prins et al., 2006a,b, 2007; Belenguer et al., 2006; Duhamel et al., 2010).

Beside the strategic aspect of depot location, a focus on tactical decisions such as vehicle routing problems (VRP) leads to the consideration of some extensions. One of these consists in integrating frequency constraints on visited customers over a given multiperiod horizon. The resulting problem is known as periodic VRP or PVRP, introduced by Christofides and Beasley (1984). As for the LRP, arc-routing versions of the problem exist (Chu et al., 2006; Lacomme et al., 2005) but most published papers consider a node routing version. The methods used to solve PVRP are mainly heuristics (Christofides and Beasley, 1984; Tan and Beasley, 1984; Chao et al., 1995). A particularly powerful approach is the tabu search algorithm proposed by Cordeau et al. (1997). Very recently, Hemmelmayr et al. (2009) have developed a variable neighborhood search heuristic which leads on average to even better results.

As special case, the PVRP can also be viewed as a multidepot VRP (MDVRP). The latter is defined on a single day but instead of visiting the customers from routes assigned to a single depot, the vehicles operate from any of the depots. Thus, by considering the routing from each depot as the routing from each period of the horizon, the statement of the MDVRP can be seen as a particular PVRP. In such a case, exact methods are available (Laporte et al., 1984; Laporte et al., 1988; Mingozzi and Valletta, 2003) and produce optimal solutions on instances involving up to 80 customers (asymmetric problem).

The LRP and the PVRP have been combined by Prodhon (2007) for the first time into an even more realistic problem: the

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periodic LRP or PLRP. Except conference papers, no publication is available on this topic. The objective is to determine the set of depots to be opened, the combination of service days to be assigned to customers and the routes originating from each depot for each period of the horizon, in order to minimize the total cost. The first approach presented in Prodhon (2007) to solve the PLRP is an iterative heuristic. Each global iteration of the algorithm is composed of three main steps: i) depot location, ii) assignment of customers to service days, iii) routing decisions. To choose the depots to open over the given horizon, the entire set of customers is considered within a single fictive day and an LRP is solved. During this first step, other information is recorded about the visited edges to be used for the second step. More precisely, the algorithm tries to gather in a same day customers most likely to succeed each other in a route from a good PLRP solution. To perform the routing, the remaining problem can be decomposed into independent MDVRP, one per day. It is solved by the Randomized Extended Clarke and Wright Algorithm proposed in Prins et al. (2006b). A local search exchanges customers' combination of service days and the algorithm handles another MDVRP according to the new assignment. This alternance is performed until convergence occurs. Then, a new global iteration begins with a diversification on the subset of open depots.

In Prodhon and Prins (2008), the proposed method of solving the PLRP is a multi-start genetic based metaheuristic that tries to take into consideration several decision levels together. Each global iteration of the algorithm begins by assigning a fixed combination of service days to each customer, with respect to their required service frequency, for the entire set of individuals from the population. During that global iteration, the evolution is tackled by a Memetic Algorithm with Population Management (MA|PM) scheme to handle the location-routing decisions. For each child, a local search occurs on the periodic aspect. This allows the recording of a possible better assignment of service days that would be used in the next global iteration of the method.

The first approaches did not focus so much on periodic decisions. Only a simple local search tried to improve this crucial issue. This paper aims to deepen the search on the assignment of customer visit days without neglecting the location-routing decisions. The proposed method manages the periodic level through an evolutionary local search (ELS). It is hybridized with a heuristic based on the Randomized Extended Clarke and Wright Algorithm (RECWA) (Prins et al., 2006b) that produces feasible PLRP solutions and thus evaluates the fitness of individuals from the ELS.

The paper is organized as follows. The problem is defined in more detail in Section 2. Section 3 describes the framework of the proposed algorithm. The performance of the method is evaluated in Section 4. Some concluding remarks close the paper.

2. Problem definition

The aim of this section is to describe formally the PLRP studied in this paper. The problem is defined on a horizon H composed of P periods (days) and a complete, weighted and undirected network $G = (V,E,C)$. V is a set of nodes comprising of a subset I of m possible depot locations and a subset $J = V\backslash I$ of n customers. C is the weight, corresponding to the travelling cost c_{ij}, associated with the set of edges E linking any two nodes i and j. A capacity W_i and an opening cost O_i are associated with each depot site $i \in I$. Each customer $j \in J$ has to be served a given number of times $s(j)$ during the horizon, and $Comb_j$ is his set of allowed combinations of service days. d_{jlr} is the demand of customer j on the day l of combination $r \in Comb_j$. A set K of N identical vehicles of capacity Q is available over H. A vehicle used at least once from a depot during the horizon incurs a fixed cost F and it may operate one single route per day. The total number of vehicles T_i used at depot i is the maximum number of routes performed during a period from depot i over H. It is a decision variable. Fig. 1 illustrates the meaning of T_i.

The following constraints must be respected:

- each customer j must be served exclusively on each day l of exactly one combination $r \in Comb_j$, by one vehicle with the amount d_{jlr};
- the number of vehicles assigned to the depots ($\sum_{i\in I} T_i$) does not exceed N;
- each route must begin and end at the same depot within the same day and its total load must not exceed the vehicle capacity;
- the total load of the routes assigned to a depot on any day $l \in H$ must fit the capacity of that depot.

The total cost of a route includes the fixed cost F and the costs of traversed edges (variable costs) on each day of the horizon. The objective is to find which subset of depots should be opened, which combination of days should be assigned to each customer and which routes should be performed, to minimize the total cost (fixed costs of depots, plus total cost of the routes).

To formulate the PLRP as a linear program, let us define $a_{rl} = 1$ iff day l belongs to serviced combination r. The following binary variables are used: $y_i = 1$ iff depot i is opened, $f_{ij} = 1$ iff customer j is assigned to depot i, $x_{ijkl} = 1$ iff edge (i,j) is traversed from i to j in the route performed by vehicle $k \in K$ on day $l \in H$, and $b_{jr} = 1$ iff visit combination $r \in Comb_j$ is assigned to customer j. The PLRP can be stated as follows:

$$\min z = \sum_{i\in I} O_i y_i + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{l\in H} c_{ij} x_{ijkl} + \sum_{i\in I} FT_i \quad (1)$$

subject to

$$\sum_{k\in K}\sum_{j\in J} x_{ijkl} \leqslant T_i \quad \forall l \in H, \quad \forall i \in I \quad (2)$$

$$\sum_{j\in V} x_{ijkl} - \sum_{h\in V} x_{hikl} = 0 \quad \forall k \in K, \quad \forall i \in V, \quad \forall l \in H \quad (3)$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijkl} \leqslant |S| - 1 \quad \forall S \subseteq J, \quad \forall k \in K, \quad \forall l \in H \quad (4)$$

$$\sum_{j\in J}\sum_{i\in V} d_{jlr} x_{ijkl} \leqslant Q \quad \forall k \in K, \quad \forall l \in H, \quad \forall r \in Comb_j \quad (5)$$

$$\sum_{j\in J} d_{jlr} f_{ij} \leqslant W_i y_i \quad \forall i \in I, \quad \forall l \in H, \quad \forall r \in Comb_j \quad (6)$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijkl} \leqslant 1 \quad \forall k \in K, \quad \forall l \in H \quad (7)$$

$$\sum_{u\in J} x_{iukl} + \sum_{u\in V\backslash\{j\}} x_{ujkl} \leqslant 1 + f_{ij} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K, \quad \forall l \in H \quad (8)$$

$$\sum_{r\in Comb_j} b_{jr} = 1 \quad \forall j \in J \quad (9)$$

$$\sum_{i\in V}\sum_{k\in K} x_{ijkl} - \sum_{r\in Comb_j} b_{jr} a_{rl} = 0 \quad \forall j \in J, \quad \forall l \in H \quad (10)$$

$$x_{ijkl} \in \{0,1\} \quad \forall i \in V, \quad \forall j \in V, \quad \forall k \in K, \quad \forall l \in H \quad (11)$$

$$y_i \in \{0,1\} \quad \forall i \in I \quad (12)$$

$$f_{ij} \in \{0,1\} \quad \forall i \in I, \quad \forall j \in J \quad (13)$$

$$b_{jr} \in \{0,1\} \quad \forall j \in J, \quad \forall r \in Comb_j \quad (14)$$

$$T_i \in \mathbb{N} \quad \forall i \in I \quad (15)$$

	Depot 1	Depot 2	Depot 3
Day 1	2	2	1
Day 2	3	2	2
Day 3	1	2	3
Day 4	2	2	2
T_i	3	2	3

Total number of vehicles used: $T_1 + T_2 + T_3 = 8$

Fig. 1. Example of the number of routes performed by depot on each day of the horizon.