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Decision Support

E-DEA: Enhanced data envelopment analysis

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ABSTRACT

Data envelopment analysis (DEA) has enjoyed a wide range of acceptance by researchers and practitioners alike as an instrument of performance analysis and management since its introduction in 1978. Many formulations and thousands of applications of DEA have been reported in a considerable variety of academic and professional journals all around the world. Almost all of the formulations and applications have basically centered at the concept of "relative self-evaluation", whether they are single or multi-stage applications. This paper suggests a framework for enhancing the theory of DEA through employing the concept of "relative cross-evaluation" in a multi-stage application context. Managerial situations are described where such enhanced-DEA (E-DEA) formulations had actually been used and could also be potentially most meaningful and useful.

1. Introduction

Almost all DEA models are based on the concept of "relative self-evaluation" for each decision making unit (DMU). There are some exceptions to this statement though. The concept of "cross-efficiency" or "peer-appraisal", in addition to "self-efficiency" or "self-evaluation", has also been used in DEA formulations, albeit in rather limiting forms. The works of Sexton et al. (1986), Oral et al. (1991, 2001), Doyle and Green (1994), Green et al. (1996), Adler et al. (2002), Wu et al. (2008, 2009), and Liang et al. (2008a,b) are some examples using the concept of "cross-efficiency" in one way or another.

The primary objective of this paper is to emphasize the usefulness and importance of using both self-evaluations and cross-evaluations properly, called enhanced-DEA, or shortly E-DEA. Conventional DEA models mostly imply that only self-evaluations scores are to be used in decision-making processes. Such an approach might prove to be limiting for some decisional contexts and one might additionally need cross-evaluations. This paper presents such decision making situations and explains how E-DEA can be used properly under these circumstances. Moreover, it will be shown that E-DEA can be integrated with non-DEA models to deal with group decision-making contexts where the issues of transparency, consensus formation, resentment avoidance, and participation are important and even required. Also compared and contrasted, through using the same actual data set, how different formulations of cross-efficiency influence the selection of R&D projects.

The structure of this paper is as follows. The next section, Section 2, introduces the concept of enhanced-DEA and explains it fully in connection with DEA, whether it is a single-stage or multi-stage modeling process. Section 3 presents one example where an E-DEA model has been constructed and used in practice. Section 4 suggests two more areas where E-DEA models can be potentially meaningful and useful. The last section, Section 5, includes some concluding remarks.

2. DEA versus E-DEA

In this section, we first present the original DEA formulation that is basically the classical "self-efficiency" model, and its use in decision making with other non-DEA models. Then we provide a formulation of E-DEA, a formulation that integrates both "self-efficiency" and "cross-efficiency" scores. Also to be discussed is the decisional context that motivates and justifies the use of E-DEA formulation, along with some non-DEA models. Another point to be made is the way the concept of "cross-efficiency" defined and used in the literature.

2.1. DEA

The basic relative performance model of DMU $-\ i$, as perceived by DMU $-\ i$ itself, can be formulated, following the CCR model (Charnes et al., 1978, 1981), as.

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2.1.1. Model A: Self-evaluation model (DEA)

$$\left\{\begin{array}{ll} E_{ii} = \text{Max} & \left(\sum_k u_{ik}y_{ik}\right) \Big/ \left(\sum_r v_{ir}x_{ir}\right) \\ \text{subject to} & \left(\sum_k u_{ik}y_{jk}\right) \Big/ \left(\sum_r v_{ir}x_{jr}\right) \leqslant 1, \quad \forall j \\ & u_{ik} \geqslant 0, \quad \forall k \text{ and } v_{ir} \geqslant 0, \quad \forall r \end{array}\right\},$$

where E_{ii} is the efficiency of DMU – i, ($i = 1,2,\ldots,n$) as "most favorably" evaluated by DMU – i, y_{jk} is the quantity of output k produced by DMU – j, $k = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$, u_{ik} is the coefficient of y_{ik}, the value of which is to be optimally determined, x_{jr} is the quantity of input r used by DMU – j, $r = 1,2,\ldots,q$ and $j = 1,2,\ldots,n$, v_{ir} is the coefficient of x_{ir}, the value of which is to be optimally determined.

Model A, in the presence of n different DMUs, needs to be used n times to estimate the self-evaluation scores of all DMUs, implying that the above optimization is to be performed n times. The self-evaluation scores, E_{ii}, are then used, either by themselves alone or combined with other methods, as it is done in the case of multi-stage applications, to make decisions. See Fig. 1.

DEA Model A provides the "most favorable" efficiency score E_{ii} for DMU – i; that is, the efficiency of DMU – i is most favorably perceived or optimistically estimated by DMU – i itself. If Model A is repeated for all DMU – i, $i = 1,2,\ldots,n$, then we have n number such optimistic estimates: E_{ii}, $i = 1,2,\ldots,n$. The Model A and the self-evaluation scores obtained from this in fact define a decision-making context with the following characteristics:

- The *efficiency* score E_{ii} is *most favorable* because of the maximization, and *relative* because of the constraints in Model A. There might be situations, however, where the concept of *most favorable* needs to be replaced by *least favorable* one (see, for instance, Oral et al., 1992; Despotis, 2002). But this need or preference does not change the very nature of DEA models.
- The set of outputs are linked non-parametrically to the set of inputs through the concept of efficiency that is expressed in ratio form. In the efficiency expression of Model A, the output coefficients (u_{ik}) and the input coefficients (v_{ir}) are to be optimally found from the perspective of DMU – i. If DMU – i is found to be *inefficient* using Model A, then managerial measures are formulated according to the efficient DMUs that are in the *reference set* of DMU – i. These efficient DMUs in the reference set can also be called *local leaders* for DMU – i. If one needs to find the efficiency of DMU – i with respect to not only to the local leaders but also with respect to a "global leader", then the formulation of Model A can be slightly modified (see, for instance, Oral and Yolalan, 1990; Oral et al., 1992; Despotis, 2002).
- The input and output coefficients (v_{ir}) and (u_{ik}), respectively, are more than being only "weights". They play two roles at the same time: (1) they convert incommensurate units into commensurate ones, and (2) they indicate the importance of inputs and outputs – only in this case they correspond to the term "weights" as used in the literature. For this double role of the coefficients, see Kettani et al. (2004).
- Letting each DMU – i determine their own optimal coefficient values, with which none of the DMUs could have an efficiency score higher than 1, in fact, defines a particular decision-making context, a context in which each DMU is allowed to have a "say" or "voice" with respect to its own relative performance. This is an important feature that DEA models are able to offer. Thus subjectivity, favoring oneself optimally, is an accepted feature and applies to every DMU equally. In a sense we can even term the efficiency scores E_{ii} as *model-based behavioral relative self-evaluations*.
- Although each DMU is allowed to have a voice with respect to its own relative performance, no DMU, however, is permitted to have a "say" or "voice" when it comes to the performance evaluations of the other DMUs in the observation set. This is rather a limiting feature of DEA models, especially for those decision contexts where one DMU's perception of the other DMUs is important and needs to be taken into consideration. In other words, DEA models produce and use only the E_{ii} values and ignore the other possible values E_{ij} of the matrix **E**, when $i \neq j$, $\forall i, j$. Here, E_{ij} is the cross-efficiency score of DMU – j from the perspective of DMU – i.

Model A implies that only the diagonal elements of a possible complete matrix $\mathbf{E} = [E_{ij}]$ are being used in "conventional" DEA

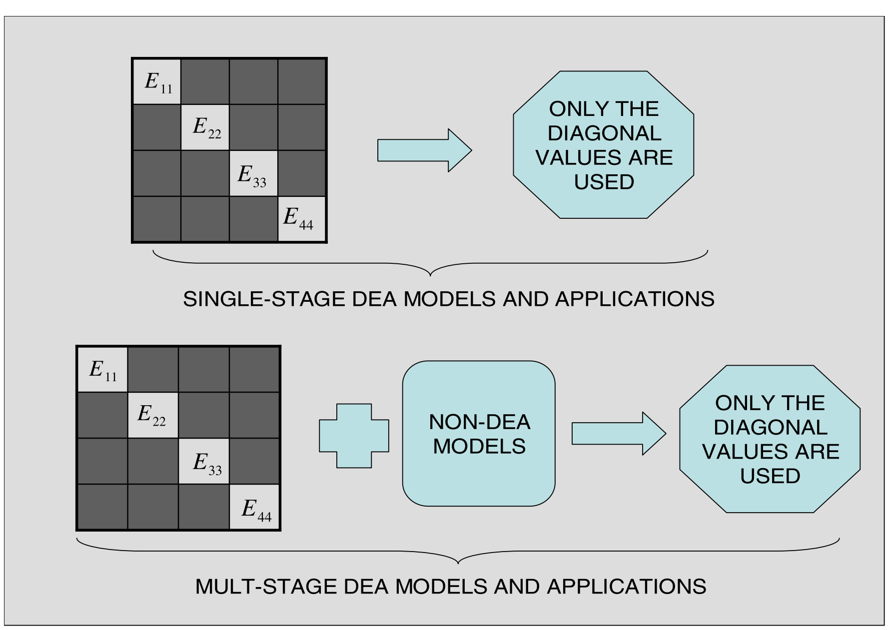

Fig. 1. Single-stage and multi-stage DEA models.