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Innovative Applications of O.R.

Computing stable loads for pallets

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ABSTRACT

This paper describes an Integer Programming model for generating stable loading patterns for the Pallet Loading Problem under several stability criteria. The results obtained during evaluation show great improvement in the number of stable patterns in comparison with results reported earlier. Moreover, most of the solved cases also ensure optimality in terms of utilization of a pallet.

1. Introduction

The Pallet Loading Problem (PLP) is defined as the problem of finding an optimal loading pattern for a set of identical boxes on a rectangular pallet. It is assumed that boxes are placed orthogonally to the edges of a pallet and that they can be rotated by 90°. Normally, the height of a box is considered to be fixed, which reduces the problem into a two-dimensional $PLP(L,W,l,w)$. With such restrictions pallet loading becomes a problem of allocating the maximal number of small rectangles of length l and width w on a bigger rectangle of length L and width W. The natural assumption is that the small rectangles do not overlap. The problem belongs to the broader class of packing, cutting and placement problems. In the typology given by Wäscher et al. (2007), PLP is classified as a two-dimensional, rectangular Identical Item Packing Problem.

Typically, the PLP arises in the area of logistics, where manufactured goods need to be packed in layers on uniform pallets for further distribution. This often requires taking into consideration some practical issues like, for example, physical stability of a load. Pallet loads are often secured by plastic wrap. However, there are situations, for example where packed boxes do not exactly fit into the area of the pallet, or other situations caused by dimension, shape or location of the load, in which plastic wrap cannot be used. The physical stability of the load can then only be obtained by arranging boxes in a special pattern. Special stable loading patterns can be used to improve stability of load secured with plastic wrap and possibly reduce amount of plastic waste.

In the current state-of-the-art methods for PLP, stability issues are either a secondary objective or not considered at all. Developers of methods that consider stability aspects often stress that there is a trade-off between high utilization of a pallet and stability of the load pattern. These methods often work in a trial-and-error fashion generating a number of patterns to find the one which satisfies stability criteria best.

Our approach is different. We show that the stability criteria may be treated as the main objective without compromising the utilization of a pallet. In contrast to previous methods, which use heuristic approaches to handle stability, our method is based on an Integer Programming (IP) formulation of the PLP and has been shown to be very effective for moderate size problems (e.g., Alvarez-Valdés et al., 2005). Furthermore, with the new method better results with respect to utilization and stability criteria are obtained on the test cases.

This paper extends an earlier study on stability of Pallet Loading Problems (Kocjan and Holmström, 2008). A new and complete Integer Programming formulation of the stability criteria is presented. The procedures for generating basic loading patterns and other computational steps differ in many details from the earlier study. Moreover, new computational results and a comparison with the state-of-the-art methods are presented.

This paper is organized as follows. Section 2 gives a short description of the previous work on stability of PLP. Section 3 describes the applied stability criteria and discuss the outline of the scheme for computing stable loads. Sections 4–6 then give in-depth descriptions of the method. Results from evaluating our method and comparison with earlier results are presented in Section 7. Finally, Section 8 summarizes the work.

2. Related work

The state-of-the-art methods for solving PLP are discussed in detail in, e.g., Alvarez-Valdés et al. (2005) and Martins and Dell

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(2008). Many of these methods apply different types of heuristics. The block-building heuristic is the most popular (see e.g., Bischoff and Dowsland, 1982; Scheithauer and Terno, 1996; Smith and de Cani, 1980; Steudel, 1979). With this heuristic, boxes are allocated on the edges of a pallet and the loading pattern is extended towards its center.

The first exact algorithm for PLP was presented in Dowsland (1987). Since then a number of exact methods have been proposed using, for instance, a branch-and-bound computing scheme (Alvarez-Valdés et al., 2005), or column generation and Lagrangian relaxation (Ribeiro and Lorena, 2007).

Stability of pallet loads was first studied by Carpenter and Dowsland (1985). These authors developed the following three stability criteria. The *supportive criterion* ensures interlocks between columns of boxes in a loading pattern. The *base contact criterion* prevents situations where a box is not supported over an arbitrary percentage of its base. Finally, the *non-guillotine criterion* prevents creating patterns with straight cuts running across the whole length or width of a pallet. The non-guillotine criterion is closely related to the supportive criterion.

The authors apply these criteria to pallet stacks created by combining layouts obtained by transforming a basic maximal pattern produced by the Bischoff–Dowsland algorithm (Bischoff and Dowsland, 1982). The transforming operations are: compacting the basic pattern by pushing individual boxes towards two adjacent edges of the pallet, reflecting patterns along the length and width of the pallet, and rotating a pattern by 180°.

Bischoff (1991) developed procedures for *compacting*, *centering blocks* and *distributing gaps* to achieve maximum stability of the loading pattern. Compacting extends the similar procedure of Carpenter and Dowsland with an additional operation preventing occurrence of jagged patterns. Centering blocks and distributing gaps are operations which reallocate boxes of a specific oriented block within its rectangular enclosure. These procedures are integrated within the algorithm of Bischoff and Dowsland (1982). As a result, the algorithm, for each specific problem, produces a greater number of possible patterns, which leads to higher probability of succeeding in combining generated patterns into a stable stack. The stability of the stack is computed in reference to the supportive and base contact criteria of Carpenter and Dowsland (1985).

Liu and Hsiao (1997a) considered a variant of PLP where boxes are stacked on their bottom, side or end surface. It is required that boxes in the same layer are placed on the same surface, creating layers of uniform height. The method developed by Liu and Hsiao (1997a) is similar to the method of Carpenter and Dowsland (1985). A basic pattern for each type of layer is generated using block-building heuristics, in this case using the method of Smith and de Cani (1980). Thereafter, for each basic pattern, three new patterns are generated using reflection and rotation. Finally, the stacking sequence is determined by combining the created patterns in such a way that the number of boxes fulfills both the supportive and base contact criteria. In a later publication (Liu and Hsiao, 1997b), the authors incorporate the compacting, block-centering and gap-distributing procedures of Bischoff (1991).

3. IP based scheme for generating stable loading patterns

Here we present the general computing scheme for generating stable patterns for pallet-loading problems. Our method incorporates the stability criteria of Carpenter and Dowsland (1985), which are the following:

- *supportive criterion:* each box must be supported by at least two boxes from the layer below, disregarding boxes with contact area less than X%,
- *base contact criterion:* at least Y% of the base area of each box is supported by boxes from the layer below,
- *non-guillotine criterion:* there is no straight or jagged cuts ($\pm I$) running across the length or width of the pallet longer than Z% of the length/width of a pallet.

The proposed scheme is based on an IP formulation of the problem and works in three stages:

Stage 1 computes the maximal number of boxes and a maximal layout for one layer with respect to non-guillotine constraints.

Stage 2 computes two layers such that both layer 1 placed on layer 2, and layer 2 placed on layer 1, fulfill the stability criteria.

Stage 3 creates full load by stacking boxes according to patterns computed in Stage 2 up to an arbitrary height H.

Stage 2, which is the core of our method, starts with computing a possible configuration of the load by reflecting/rotating the obtained pattern. If any of the combinations of generated patterns fulfills the stability criteria for both layers, in any way they are combined, then the complete load is generated using the obtained combination of layers. If none of the configurations fulfill such criteria, a new stable model for two-layer PLP, extended with stability criteria, is computed.

Details on both methods are given in two later sections, Sections 5 and 6.

4. Computing non-guillotine PLP

The optimal layout of the layer is computed using an IP formulation of the PLP adopted from Beasley (1985) and extended with constraints preventing guillotine cuts. The formulation uses two types of 0–1 variables, h_{ij} and v_{ij}. They equal 1, if a box is placed horizontally and vertically, respectively, with their lower left corner in position (i,j). The IP formulation of the problem is

$$\max \quad \sum_{i=0}^{L-l}\sum_{j=0}^{W-w} h_{ij} + \sum_{i=0}^{L-w}\sum_{j=0}^{W-l} v_{ij}, \tag{1}$$

$$\text{subject to} \quad \sum_{i=\max\{0,r-l\}}^{\min\{r,L-l\}}\ \sum_{j=\max\{0,s-w\}}^{\min\{s,W-w\}} h_{ij} + \sum_{i=\max\{0,r-w\}}^{\min\{r,L-w\}}\ \sum_{j=\max\{0,s-l\}}^{\min\{s,W-l\}} v_{ij} \leqslant 1, \tag{2}$$

$$(r=0,\ldots,L-1;\ s=0,\ldots,W-1),$$

$$\sum_{i=\max(1,p-l)}^{\min(p+l+I,L-1)}\ \sum_{j=a}^{\min(a+\lfloor W\cdot Z\rfloor,W)} h_{ij} + \sum_{i=\max(1,p-l)}^{\min(p+w+I,L-1)}\ \sum_{j=a}^{\min(a+\lfloor W\cdot Z\rfloor,W)} v_{ij} \geqslant 1,$$

$$(a=0,\ldots,W;\ p=0,\ldots,L-1), \tag{3}$$

$$\sum_{i=\max(1,q-l)}^{\min(q+l+I,W-1)}\ \sum_{j=b}^{\min(b+\lfloor L\cdot Z\rfloor,L-1)} h_{ij} + \sum_{i=\max(1,q-l)}^{\min(q+w+I,W-1)}\ \sum_{j=b}^{\min(b+\lfloor L\cdot Z\rfloor,L)} v_{ij} \geqslant 1,$$

$$(b=0,\ldots,L;\ q=0,\ldots,W-1), \tag{4}$$

$$h_{ij}\in\{0,1\}\ (0\leqslant i\leqslant L-l;\ 0\leqslant j\leqslant W-w), \tag{5}$$

$$v_{ij}\in\{0,1\}\ (0\leqslant i\leqslant L-w;\ 0\leqslant j\leqslant W-l). \tag{6}$$

Constraints (2) ensure that no boxes will overlap, and are often referred to as *cover constraints*. Constraints (3) and (4) prevent an occurrence of any straight or jagged ($\pm I$) guillotine cuts traversing more than Z% of the maximum length or width of the pallet. They state that any straight line of length $Z \cdot L$ and $Z \cdot W$ or a jagged line $Z \cdot L \pm I$ and $Z \cdot W \pm I$ orthogonal to edges of the pallet, except the ones on its very edges, must have at least one box placed on it in such a way that the line itself lies between, and not directly under,