European Journal of Operational Research 206 (2010) 350–360

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

The multi-product newsboy problem with supplier quantity discounts and a budget constraint

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ARTICLE INFO

Article history:
Received 23 December 2008
Accepted 27 February 2010
Available online 6 March 2010

Keywords:
Inventory
Newsboy model
Quantity discount
Budget constraint
Lagrangian relaxation
Mixed integer nonlinear programming

ABSTRACT

This paper considers the multi-product newsboy problem with both supplier quantity discounts and a budget constraint, while each feature has been addressed separately in the literature. Different from most previous nonlinear optimization models on the topic, the problem is formulated as a mixed integer nonlinear programming model due to price discounts. A Lagrangian relaxation approach is presented to solve the problem. Computational results on both small and large-scale test instances indicate that the proposed algorithm is extremely effective for the problem. An extension to multiple constraints and preliminary computational results are also reported.

1. Introduction

The newsboy problem, also known as newsvendor problem or single period problem, has a rich history. Significant number of articles has been published to address the variants or extensions of the classical newsboy problem due to its importance to both inventory theory and practice. An important extension of the problem is to address multi-product newsboy problem with budget or other constraints (see, e.g., Hadley and Whitin, 1963; Lau and Lau, 1996). Another recent extension of the problem is to consider price discount in newsboy models, which is a popular policy used by suppliers to promote their products (see, e.g., Khouja, 1996). Though a budget constraint and supplier price discounts are two common characteristics in many newsboy circumstances, very few works consider both in a newsboy model. In this paper, we extend the multi-product newsboy problem to the situation that the newsboy faces both a budget constraint and quantity discounts offered by suppliers. The problem is to find multiple products' order quantities that take the advantage of the price discount and maximizes the expected profit under the budget constraint.

Considerable research has been devoted to analyzing multi-product newsboy problems with constraints. Among those work, Hadley and Whitin (1963) were the first to consider single constraint multi-product newsboy problem. Lau and Lau (1995, 1996) extended the constrained newsboy problem to handle general demand distributions, and developed effective solution procedures for both single- and multi-constraint cases. Vairaktarakis (2000) considered robust newsboy models with a budget constraint. The models are applicable when demand uncertainty can be described using discrete or interval scenarios. Erlebacher (2000) presented optimal and heuristic solutions for a multi-item newsvendor problem with a single capacity constraint. Three effective heuristics for the problem were developed. Moon and Silver (2000) considered a multi-item newsvendor problem subject to a budget constraint and fixed ordering costs. A dynamic programming approach and a heuristic were presented for small- and large-scale instances, respectively. Silver and Moon (2001) studied the multi-item single period problem with an initial stock of units that can be converted into end items. Abdel-Malek et al. (2004) observed that there is a renewed interest in inventory control analysis with today's emphasis on supply chain management. They proposed models for solving the multi-product newsboy problem with a budget constraint, and provided different solution formulae for uniform, exponential, and other probability demand distributions. Abdel-Malek and Montanari (2005) analyzed the solution space for a multi-product newsboy problem with a budget constraint and presented a solution method. Following their previous work, Abdel-Malek et al. (2008) recently extended the multi-product newsboy model with a budget constraint to cover random yield scenarios, designated as the Gardener Problem. They developed solution methodologies based on the application of Lagrange multipliers, Leibniz's rule and Newton's method.

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doi:10.1016/j.ejor.2010.02.038

Price discount has long been used in practice as an effective promotion policy to encourage the buyers to purchase more and to achieve economies of scale. Jucker and Rosenblatt (1985) considered three types of quantity discounts in single period inventory model, and presented marginal-cost solution procedure. They argued that the behavioral implications of the all-units discount schedule are more complex and interesting than the literature has suggested. Guder et al. (1994) addressed capacitated multiple-item ordering with incremental quantity discounts for deterministic demands, and presented Lagrangian relaxation heuristic. Khouja (1995) extended the single-product newsboy problem to a case that multiple discounts are progressively used to sell excess inventory. Two models with different objectives were developed. Khouja (1996) was the first to consider both supplier quantity discounts and retailer multiple discounts in a newsboy model. A single-product newsboy problem with all-unit quantity discounts was formulated and solved. Khouja and Mehrez (1996) formulated a multi-product newsboy problem in which the newsboy uses multiple discounts to sell excess inventory under a storage constraint. They focused on the special case where the additional quantity of a product that can be sold at a discount is directly proportional to the realized demand at the regular price, and provided an efficient algorithm to solve the problem with the assumption. The problem did not involve supplier price discounts, which are not controlled by the newsboy. Lin and Kroll (1997) presented models with dual performance measure for single-item newsboy problem with quantity discount. Khouja (2000) extended the newsvendor problem to the case in which demand is price-dependent and multiple discounts with prices under the control of the newsvendor are used to sell excess inventory. Ji and Shao (2006) addressed bilevel newsboy problem with fuzzy demands and discounts without budget or other constraints, where the manufacturer gives the retailers either all-unit discounts or incremental discount.

Both a budget constraint and suppliers price discounts are very common in supplier-retailer practice, but to our knowledge, the newsboy model with taking into account both features has not been reported in the literature. Typically the optimal order quantity depends on the purchasing price (and other parameters), which is fixed in previous research. However, a more realistic situation is that the price is also dependent on the order quantity. Under demand uncertainty and the budget constraint, how to allocate resource to each product to take the advantage of supply discounts and also avoid overstock cost is a challenging problem. From the optimization point of view, as shown in Section 2, due to taking into account discounts, the problem is formulated as a mixed integer nonlinear programming model. The previous constrained newsboy models are nonlinear programming, and most of them can be viewed as a special case of our model without price discounts. Comparing with those nonlinear programming newsboy models, solving the proposed problem is much harder, especially for large-scale instances.

The purpose of this study is to investigate the affect of both a budget constraint and supplier quantity discounts on the optimal order quantities in a multi-product newsboy problem. In the next section, we present a mixed integer nonlinear programming model to formulate the problem. A Lagrangian relaxation approach is proposed in Section 3. Section 4 reports the computational results. Section 5 discusses an extension to multiple constraints. Finally, Section 6 presents the conclusions and some future work.

2. The model

Assume that suppliers provide all-unit quantity discounts, and the newsboy has a budget constraint and faces uncertain demands for multiple products. We also assume the uncertainty information of the demand, such as probability density function for each product, is given.

The following notations are used in the formulation of the problem:

Indices

$i = 1, \ldots, n$	index of products, where n is the total number of products
k_i	the number of quantity discounts for product i offered by a supplier
$j = 1, \ldots, k_i$	index of discount segment j for product i offered by a supplier

Parameters

p_i	unit sales revenue of product i
h_i	the resource consumed per unit of product i
H	the budget limitation of the newsvendor
c_{ij}	the unit discounted price of product i on discount segment j, and c_{i1} is original unit price
d_{ij}^L	the lower bound of the order quantity of product i on discount segment j
d_{ij}^U	the upper bound of the order quantity of product i on discount segment j
z_i	the demand for product i, a random variable
$f_i(z_i)$	the probability density function followed by the demand of product i
g_i	the unit understocking cost of product i
s_i	the unit overstocking cost (excluding purchasing cost) of product i. s_i can be negative if there is salvaging value for overstocking items instead of cost. In that case, $-s_i$ is unit salvaging value of product $i (-s_i \leqslant c_{ij})$

We have the following discount relationships

$$d_{ij-1}^L < d_{ij-1}^U < d_{ij}^L < d_{ij}^U, \quad d_{i1}^L = 0, \text{ for all } i, \text{ and } c_{ij-1} > c_{ij}, \text{ for all } i, \text{ and } j = 2, \ldots, k_i.$$

Define the following decision variables

Q_i	the quantity of product i purchased from suppliers
Q_{ij}	the quantity of product i purchased at price discount segment j
y_{ij}	1 if product i is purchased at price segment j; otherwise 0

Thus, the objective of maximizing the expected benefit is formulated as:

$$\text{Maximize} \quad R = \sum_{i=1}^{n} \left\{ \int_0^{Q_i} [z_i p_i - s_i(Q_i - z_i)] f_i(z_i) dz_i + \int_{Q_i}^{\infty} [p_i Q_i - g_i(z_i - Q_i)] f_i(z_i) dz_i \right\} - \sum_{i=1}^{n} \sum_{j=1}^{k_i} c_{ij} Q_{ij} \tag{1}$$