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# Robust production planning in a manufacturing environment with random yield: A case in sawmill production planning

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ABSTRACT

This paper addresses a multi-period, multi-product sawmill production planning problem where the yields of processes are random variables due to non-homogeneous quality of raw materials (logs). In order to determine the production plans with robust customer service level, robust optimization approach is applied. Two robust optimization models with different variability measures are proposed, which can be selected based on the tradeoff between the expected backorder/inventory cost and the decision maker risk aversion level about the variability of customer service level. The implementation results of the proposed approach for a realistic-scale sawmill example highlights the significance of using robust optimization in generating more robust production plans in the uncertain environments compared with stochastic programming.



## 1. Introduction

Production planning in many manufacturing environments is based on some parameters with uncertain values. Uncertainties might arise in product demand, yield of processes, etc. Thus, the robustness of a production plan, in term of fulfillment of product demand, is dependent on incorporating the uncertain parameters in production planning models.

This study is concentrated on multi-period, multi-product (MPMP) production planning in the sawing units of sawmills, where possible combinations of log classes and cutting patterns produce simultaneously different mix of lumbers. As logs are grown under uncertain natural circumstances, non-homogeneous and random characteristics (in terms of diameter, number of knots, internal defects, etc.) can be observed in different logs in each class. Consequently, the processes yields (quantities of lumbers that can be produced by each cutting pattern) are random variables. Lumber demand in each period is assumed as a deterministic parameter which is determined based on the received orders. That is, we do not deal with "seasonality" or "trend-based" demand in this work. In the sawmill production planning problem, we are looking for the optimal combination of log classes and cutting patterns that best fit against lumber demand. The part of demand that cannot be fulfilled on time, due to machine capacities and/or uncertain yield, will be postponed to the next period by considering a backorder cost. The objective is to minimize products inventory and backorder costs and raw material consumption cost, regarding fulfillment of product demand, machine capacities, and raw material (log) inventory. The uncertainty in the yields of cutting patterns in sawmills can be represented as uncertain yield coefficients in the coefficients of constraints matrix. Regarding to the potential significance of yield uncertainty on the production plan, and customer orientation which is at center of attention in the sawmills which are dependent on the export markets, obtaining robust plans with minimum backorder size (service level) variability is an important goal of production planning in sawmills.

Sawmill production planning problem can be considered as the combination of several classical production planning problems in the literature which have been modeled by linear programming (LP). This problem was formulated by a deterministic LP model and was solved based on the average values for processes yields in Gaudreault et al. (2004). However, if decisions are made based on the deterministic model, there is a risk that the demand might not be met with the right products. Consequently, it results high inventory levels of products with low quality and price as well as extra levels of backorder of products with high quality and price (decreased customer service level). The other approach in the literature for sawmill production planning is focused on combined optimization type solutions linked to real-time simulation sub-systems (Maness and Norton, 2002; Maness and Adams, 1991; Mendoza et al., 1991). In this approach, the stochastic characteristics

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of logs are taken into account by assuming that all the input logs are scanned through an X-ray scanner, before planning. Maness and Norton (2002) developed an integrated multi-period production planning model which is the combination of an LP model and a log sawing optimizer (simulator). The LP model acts as a coordinating problem that allocates limited resources. The log sawing optimization models are used to generate columns for the coordinating LP based on the products' shadow prices. Although the stochastic characteristics of logs are considered in this approach, it includes the following limitations: logs, needed for the next planning horizon, are not always available in sawmills to be scanned before planning. Furthermore, to implement this method, the logs should be processed in production line in the same order they have been simulated, which is not an easy practice. Finally scanning logs before planning is a time consuming process in the high capacity sawmills which delays the planning process.

There are several techniques to incorporate uncertainty in optimization models, including stochastic programming (Kall and Wallace, 1994; Birge and Louveaux, 1997; Kall and Mayer, 2005), and robust optimization (Mulvey et al., 1995). Bakir and Byrne (1998) developed a stochastic LP model based on the two-stage deterministic equivalent problem to incorporate demand uncertainty in a multi-period multi-product (MPMP) production planning model. In Escudero et al. (1993) a multi-stage stochastic programming approach was proposed for solving a MPMP production planning model with random demand. Kazemi et al. (2007b, 2008) proposed a two-stage stochastic model for sawmill production; it was shown in Kazemi et al. (2007b, 2008) that the proposed production plans by stochastic programming approach results a considerably lower expected inventory and backorder cost than the plans of the mean-value deterministic model. It is important to note that stochastic programming approach focuses on optimizing the expected performance (e.g. cost) over a range of possible scenarios for the random parameters. We can expect that the system would behave optimally in the *mean* sense. However, the system might perform poorly at a particular realization of scenarios such as the worst-case scenario. More precisely, unacceptable inventory and backorder size for some scenarios might be observed by implementing the solution of two-stage stochastic model. To handle the tradeoff associated with the expected cost and its variability in stochastic programs, Mulvey et al. (1995) proposed the concept of robust optimization. Leung and Wu (2004) proposed a robust optimization model for stochastic aggregate production planning. In Leung et al. (2007) a robust optimization model was developed to address a multi-site aggregate production planning problem in an uncertain environment. In Kazemi et al. (2007a) robust optimization approach was proposed as one of the potential methodologies to address MPMP production planning in a manufacturing environment with random yield.

In this paper, a robust optimization (RO) approach is proposed for multi-period sawmill production planning while considering random characteristics of raw materials (logs) and consequently random processes yields. The random yields are modeled as scenarios with a stationary discrete probability distribution during the planning horizon. We are studying a service sensitive company that wants to establish a reputation for always meeting customer service level. We also define the customer service level as the proportion of the customer demand that can be fulfilled on time, and we use the expected backorder size as a measure for evaluating the service level. Thus, the need for robustness has been mainly recognized in term of determining a robust customer service level by minimizing the products backorder size variability in the presence of different scenarios for random yields. The robustness in the products inventory size is also considered in this problem. Two alternative variability measures are used in the robust optimization model which can be selected depending on risk aversion level of decision maker about backorder/inventory size variability and the total cost. The proposed robust optimization (RO) approach is applied for a realistic-scale sawmill production planning example. The resulted large-scale quadratic programming models are solved by CPLEX 10 in a reasonable amount of time. A comparison between the backorder/inventory size variability in the two-stage stochastic model and the two robust optimization models is provided. Finally, the tradeoff between the backorder/inventory size variability and the expected total cost in the two RO models is discussed and a decision framework to select among them is proposed.

The main contributions of this paper can be summarized as follows. Applying robust optimization approach as a robust tool for sawmill production planning, regarding to the limitations of the existing approaches for sawmill production planning; comparing the performance of two different robust optimization models in controlling the robustness of production plans through applying them for a prototype sawmill; proposing a framework for selecting the most appropriate robust optimization model depending on the risk preferences of the decision maker about service level robustness and total expected cost of plans.

The rest of this paper is organized as follows. In Section 2, sawmill processes and specific characteristics are introduced. In Section 3, the robust optimization formulation for two-stage stochastic programs is provided. In Section 4, the proposed robust optimization model for multi-period sawmill production planning is presented. In Section 5, the scenario generation approach for random yields is described. In Section 6, the computational results of implementing the proposed robust optimization models for a prototype sawmill are provided. Our concluding remarks are given in Section 7.

## 2. Sawmill processes and specific characteristics

There are a number of processes that occur at a sawmill: log sorting, sawing, drying, planing and grading (finishing). Raw materials in the sawmills are the logs which are transported from different districts of forest after bucking the felled trees. The finished and graded lumbers (products) are then transported to the domestic and international markets. Fig. 1 illustrates the typical processes. In this paper we focus on operational level production planning in the sawing units of sawmills. In the sawing units, logs are classified according to some attributes namely: diameter class, species, length, taper, etc. Logs are broken down into different dimensions of lumbers by means of different cutting patterns. See Fig. 2 for three different cutting patterns. Each cutting pattern is a combination of activities that are run on a set of machines. From each log, several pieces of sawn lumber (e.g. 2(in)-4(in) $\times$ 8(ft), 2(in) $\times$ 4(in) $\times$ 10(ft), 2(in) $\times$ 6(in) $\times$ 16(ft),...) are produced depending on the cutting pattern. The lumber quality (grade) as well as its quantity yielded by each cutting pattern depends on the quality and characteristics of the input logs. Despite the classification of logs in sawmills, a variety of characteristics might be observed in different logs in each class. In fact, natural variable conditions that occur during the growth period of trees make it impossible to anticipate the exact yields of a log. As it is not possible in many sawmills to scan the logs before planning, the exact yields of cutting patterns for different log classes cannot be determined in priori.

## 3. Robust optimization formulation for two-stage stochastic programs

The robust optimization method developed by Mulvey et al. (1995) extends stochastic programming by replacing traditional