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Discrete Optimization

New solution methods for single machine bicriteria scheduling problem: Minimization of average flowtime and number of tardy jobs ☆

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ABSTRACT

We consider the bicriteria scheduling problem of minimizing the number of tardy jobs and average flowtime on a single machine. This problem, which is known to be NP-hard, is important in practice, as the former criterion conveys the customer's position, and the latter reflects the manufacturer's perspective in the supply chain. We propose four new heuristics to solve this multiobjective scheduling problem. Two of these heuristics are constructive algorithms based on beam search methodology. The other two are metaheuristic approaches using a genetic algorithm and tabu-search. Our computational experiments indicate that the proposed beam search heuristics find efficient schedules optimally in most cases and perform better than the existing heuristics in the literature.

1. Introduction

Many existing studies on scheduling consider the optimization of a single objective. In practice, however, there are situations in which a decision maker evaluates schedules with respect to more than one measure. Several recent multicriteria scheduling papers address single machine bicriteria scheduling problems. In the vein of this literature, the current study considers the minimization of mean flowtime ($\overline{F}$) and the number of tardy jobs (n_T) on a single machine. Our contribution lies in developing new heuristics that outperform the current approximate solution methodologies and in characterizing the effectiveness of these proposed heuristics in terms of various problem parameters.

The number of tardy jobs and average flowtime are significant criteria for characterizing the behavior of manufacturers who want to meet the due dates of their customers while minimizing their own inventory holding costs. The solution to the single machine problem can be used as an aggregate schedule for the manufacturer, or for generating a more detailed schedule for a factory based on a bottleneck resource. We propose four heuristics to find approximately the *efficient schedules* that minimize n_T and $\overline{F}$ on a single machine. *Efficient schedules* are the set of schedules that cannot be dominated by any other feasible schedule. All other schedules that are not in this set are dominated by at least one of these efficient schedules. Although optimizing either of the objectives, n_T or $\overline{F}$, on a single machine is polynomially solvable, finding efficient schedules that account for them simultaneously is NP-hard (Chen and Bulfin, 1993).

In the literature, most studies on bicriteria scheduling consider a single machine and the minimization of couples of criteria, such as the following: maximum tardiness and flowtime (Smith, 1956; Heck and Roberts, 1972; Sen and Gupta, 1983; Köksalan, 1999; Lee et al., 2004; Haral et al., 2007), maximum earliness and flowtime (Köksalan et al., 1998; Köktener and Köksalan, 2000; Köksalan and Keha, 2003), maximum earliness and number of tardy jobs (Güner et al., 1998; Kondakci et al., 2003), total weighted completion time and maximum lateness (Steiner and Stephenson, 2007), and total earliness and tardiness (M'Hallah, 2007). Extensive surveys of bicriteria single machine scheduling studies are provided by Dileepan and Sen (1988), Fry et al. (1989), and Yen and Wan (2003). Several recent papers investigate bicriteria scheduling problems in other machining environments (Allahverdi, 2004; Toktaş et al., 2004; Arroyo and Armentano, 2005; Gupta and Ruiz-Torres, 2005; Varangharajan and Rejendran, 2005; Vilcot and Billaut, 2008). Nagar et al. (1995), T'kindt and Billaut (1999), and Hoogeveen (2005) review the multicriteria scheduling literature. Other notable studies on multicriteria scheduling investigate the complexity of several problems (e.g., Chen and Bulfin, 1993; T'kindt et al., 2007).

Chen and Bulfin (1993) report that the problem of minimizing n_T while $\overline{F}$ is optimum, on a single machine, can be optimally solved by a polynomial time algorithm, a.k.a. the *adjusted SPT order*. This algorithm uses Moore's Algorithm on the SPT order to break

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ties among jobs with equal processing times; we refer to the sequence generated according to this algorithm as the *SPT order*. In another study, Emmons (1975) develops an algorithm for the problem of minimizing $\overline{F}$ while n_T is optimum, which is shown to be NP-Hard by Huo et al. (2007).

In the current paper, we seek efficient schedules to minimize the number of tardy jobs and average flowtime on a single machine. The first study on this problem was by Nelson et al. (1986), who proposed a constructive heuristic and an optimal solution based on a branch and bound procedure. In another study, Kiran and Unal (1991) define several characteristics of the efficient solutions. Kondakci and Bekiroglu (1997) present some dominancy rules, which they use to improve the efficiency of the optimal solution procedure by Nelson et al. (1986). Recent studies on the problem propose some general-purpose procedures. Köktener and Köksalan (2000) and Köksalan and Keha (2003) developed heuristic methods based on simulated annealing and a genetic algorithm, respectively. The latter study reports that a genetic algorithm generally outperforms simulated annealing in terms of solution quality, however, a simulated annealing approach is faster than a genetic algorithm.

After reviewing these studies, we observe that only a few solution methodologies (one exact and three heuristics) were proposed for the problem considered in this paper. Moreover, these solution methods are not compared with each other in detail. The only exception is a study by Köksalan and Keha (2003), in which the authors test the performance of their proposed genetic algorithm, relative to the simulated annealing approach of Köktener and Köksalan (2000). A comparison of these two iterative methods with respect to the optimum solution was also made, however, it was limited to a problem size of 20 jobs. In this study, we present four new algorithms: two are constructive algorithms, based on the beam search method, and the other two work iteratively utilizing a genetic algorithm (GA) and tabu-search (TS). We compare these proposed heuristics with each other and with the exact and heuristic solution methods available in the literature.

The organization of this paper is as follows: In Section 2, we present an explicit mathematical formulation for the problem of minimizing the number of tardy jobs and average flowtime on a single machine. In Section 3, we describe Nelson et al.'s (1986) optimal solution method for this problem. The proposed beam search algorithms are presented in Section 4, and GA and TS algorithms are described in Section 5. We discuss the findings of our extensive numerical study in Section 6. Finally, we present general conclusions and future research directions in Section 7.

2. Problem formulation

We consider a single machine environment in which N jobs are to be scheduled with the objective of minimizing the number of tardy jobs and average flowtime. In this environment, jobs have due dates and deterministic processing times. We assume that preemption is not allowed and that there exists no precedence relationship between jobs. P_j and d_j are the processing time and the due date of job j, respectively. Denoting S as a feasible schedule, $\overline{F}(S)$ represents the average flowtime of schedule S, and $n_T(S)$ refers to the number of tardy jobs resulting from schedule S.

Our approach aims at finding efficient schedules for minimizing $\overline{F}$ and n_T. More formally, we are interested in finding a set of schedules where, if S is an element of this set, then there exists no schedule S' satisfying the following constraints, while at least one of these constraints is strict:

$$n_T(S') \leqslant n_T(S), \quad \text{and} \quad \overline{F}(S') \leqslant \overline{F}(S).$$

The solution approach builds on the fact that optimizing either one of the objectives, n_T or $\overline{F}$, on a single machine is polynomially solvable. It is well known in the scheduling literature that the shortest processing time (SPT) rule minimizes the average flowtime and that Moore's Algorithm (Moore, 1968) minimizes the number of tardy jobs. In the rest of the manuscript, we will denote n_T (SPT) and n_T (Moore) as the number of tardy jobs when all jobs are sequenced using the SPT rule and Moore's Algorithm, respectively. Kiran and Unal (1991) showed that for each number of tardy jobs between n_T (SPT) and n_T (Moore), there exists at least one corresponding efficient schedule. The range between n_T (SPT) and n_T (Moore) is referred to as the *efficient range* of the number of tardy jobs. Any schedule having a number value of tardy jobs that is outside the efficient range is dominated by some efficient schedule. Since there exists at least one efficient schedule for every n_T value in this range, the total number of efficient schedules for a given problem is at least n_T (SPT) – n_T (Moore) + 1. Therefore, for a problem with N jobs, we solve the following model for all n such that n_T (SPT) $\geqslant n \geqslant n_T$ (Moore).

$$\begin{aligned}&\text{Min}_{\forall S} \quad \overline{F}(S)\\&\text{s.t.} \qquad n_T(S) = n.\end{aligned}$$

To present a more detailed formulation of the above problem, let us define X_{ij} and Y_j as follows:

$$X_{ij} = \begin{cases} 1, & \text{if } i\text{th position is held by job } j \\ 0, & \text{o.w.} \end{cases} \quad \text{and}$$

$$Y_j = \begin{cases} 1, & \text{if job } j \text{ is tardy} \\ 0, & \text{o.w.} \end{cases}$$

Also, let M and ξ denote a very large and very small number, respectively. We next present an explicit mathematical model for our problem. Recall that this model should be solved for all $n \in [n_T$ (Moore), n_T (SPT)]

$$\text{Min} \quad \frac{1}{N}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}(N-i+1)X_{ij}P_j\right)$$

$$\text{s.t.} \quad \sum_{j=1}^{N} X_{ij} = 1 \quad \text{for all } i \in \{1,2,\dots N\}, \tag{1}$$

$$\sum_{i=1}^{N} X_{ij} = 1 \quad \text{for all } j \in \{1,2,\dots N\}, \tag{2}$$

$$d_j - P_j - \sum_{r=2}^{N}\sum_{i=1}^{r-1}\sum_{k=1}^{N} X_{rj}X_{ik}P_k \geqslant -M \times Y_j \quad \text{for all } j \in \{1,2,\dots N\}, \tag{3}$$

$$d_j - P_j - \sum_{r=2}^{N}\sum_{i=1}^{r-1}\sum_{k=1}^{N} X_{rj}X_{ik}P_k \leqslant M \times (1-Y_j) - \xi \quad \text{for all } j \in \{1,2,\dots N\}, \tag{4}$$

$$\sum_{j=1}^{N} Y_j = n. \tag{5}$$

In the above formulation, Eq. (1) assures that only one job can be assigned to each position in the schedule. Eq. (2) makes sure that there is no unassigned job. Expressions (3) and (4) jointly identify whether job j is tardy or not, i.e., $Y_j = 0$ or $Y_j = 1$. Finally, Eq. (5) states that only n jobs are tardy. Inequalities (3) and (4) are nonlinear, due to the multiplication of X_{rj} and X_{ik}. Since both variables are binary, however, it is possible to linearize these inequalities by replacing $X_{rj}X_{ik}$ with Z_{rjik} and adding the following three constraints to the model for all $i, j, k, r \in \{1,\dots,N\}$.

a) $X_{rj} \geqslant Z_{rjik}$, b) $X_{ik} \geqslant Z_{rjik}$, c) $Z_{rjik} \geqslant X_{rj} + X_{ik} - 1$.

Observe that, the efficient schedule that has n_T (SPT) tardy jobs is the schedule that is formed according to the SPT order. Therefore, the remaining n_T (SPT) – n_T (Moore) efficient schedules need to be