European Journal of Operational Research 201 (2010) 183–193

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

On the system optimum of traffic assignment in $M/G/c/c$ state-dependent queueing networks

F.R.B. Cruz [a,*,1], T. van Woensel [b], J. MacGregor Smith [c], K. Lieckens [d]

[a] School of Computer Science and Information Technology, The University of Nottingham, Jubilee Campus, Wollaton Road, Nottingham, NG8 IBB, UK
[b] Department of Technology Management, Operations, Planning, Accounting and Control, Eindhoven University of Technology, Eindhoven, The Netherlands
[c] Department of Mechanical and Industrial Engineering, University of Massachusetts, Amherst, MA 01003, USA
[d] Department of Applied Economics, University of Antwerp, Antwerp, Belgium

ARTICLE INFO

Article history:
Received 19 April 2007
Accepted 3 March 2009
Available online 20 March 2009

Keywords:
System optimum
Traffic assignment
Queueing networks
Genetic Algorithms

ABSTRACT

The classical Wardrop System Optimum assignment model assumes that the users will cooperate with each other in order to minimize the overall travel costs. The importance of the system optimum model lies on its well-recognized ability of producing solutions that correspond to the most efficient way of using the scarce resources represented by the street and road capacities. In this paper, we present a version of the system optimum model in which the travel costs incurred on each path come from $M/G/c/c$ state-dependent queueing networks, a stochastic travel time estimation formula which takes into account congestion effects. A Differential Evolution algorithm is proposed to solve the model. We motivate this version of the problem in several ways and computational results show that the proposed approach is efficient.

1. Introduction

There have been successful attempts in the literature to model how users select their route in a congested network (for instance, see Helbing et al., 2005 and references therein). Two major streams of work can be distinguished: the system optimum (SO) models versus the User Equilibrium (UE) models.

The User Equilibrium model, assuming perfect knowledge of the travel costs, states that drivers will choose the best route according to Wardrop's first principle. This principle is equivalent to a mixed-strategy Nash equilibrium of an n-player, non-cooperative game (Bell and Cassir, 2002). The Deterministic User Equilibrium (UE) is an important classical traffic assignment model approach (Sheffi, 1985), which even recently keeps receiving improvements (see, for instance, the recent paper by Watling (2006)). In equilibrium, routes carrying a positive flow will have equal travel costs. The disadvantage of the User Equilibrium model is that the scarce resources (street and road capacity) may be used in an inefficient way (Helbing et al., 2005).

In contrast, the classical Wardrop System Optimum (SO) assignment model, assumes that all users are able to cooperate with each other in order to minimize the overall system-wide travel costs (Sheffi, 1985). Even though the system optimum (SO) assignment model is based on a rather non-realistic behavioral assumption, we argue that its solution may be seen as a result of a well-succeeded control action on the transportation network, such as, for instance, by route inducement (Moreno-Quintero, 2006). In other words, signal timings may be re-optimized and alternative routes may be re-defined in response to an increase in demand. It is well known that traffic lights and adaptive routing can improve the flow (Poli et al., 2005), depending on the traffic densities (e.g. using DRIPS, Dynamic Routing Information Panel Systems). Next to this, several paradoxes show the deficiency of the UE optimum compared to the SO model. For example, Braess's paradox shows that adding extra capacity to a network, when people selfishly choose their own route, can reduce overall performance (Braess, 1968; Braess et al., 2005). A similar result has been observed by Sheffi and Daganzo (1978). On the other hand, Charnes and Klingman (1971) showed that both increasing supply and demand could counter-intuitively lead to a reduction in total costs. In any case, both paradoxes show that transport planners should not trust in the users' selfish actions when optimizing the traffic network. As such, these paradoxes enforce the necessity of the system optimum (SO) assignment model.

One major problem in the above models is that the travel times are usually assumed to be either deterministic or approximate stochastic models. Typically, the SO models express the travel costs in terms of deterministic travel time functions (Prashker and Bekhor, 2000), yet these times are known to be rather variable between trips, within and between days. The relevant travel time models

* Corresponding author. Tel.: +55 31 3409 5929; fax: +55 31 3409 5924.
E-mail addresses: frc@cs.nott.ac.uk (F.R.B. Cruz), T.v.Woensel@tm.tue.nl (T. van Woensel), jmsmith@ecs.umass.edu (J. MacGregor Smith), Kris.Lieckens@ua.ac.be (K. Lieckens).
[1] On sabbatical leave from the Department of Statistics, Federal University of Minas Gerais, Belo Horizonte, MG, Brazil.

0377-2217/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2009.03.006

are usually built on the classical formulas that have been constructed over the past 40 years. For instance, the well-known BPR (Bureau of Public Roads, 1964) was developed in 1964 using data from the Highway Capacity Manual.

Kimber and Hollis (1979) developed another travel time formula based upon an approximation to the time-dependent $M/G/1/\infty$ model. Since analytical expressions for the transient $M/G/1/\infty$ model are intractable, they developed an approximation based upon a coordinate transformation technique to adjust the steady-state formula to account for the transient effects of the queue. In their approach, they can account for existing traffic on the highway link, but they fix the service rate of the traffic link μ, the queue is infinite in capacity, and there is only one server for the traffic. Subsequently Akçelik (1991) extended the work of Kimber and Hollis with formulas based upon the coordinate transformation technique that are recognized as efficient to model the travel times, especially under congestion during rush hours, when the demand far exceeds capacity (Ceylan and Bell, 2005). The performance of Akçelik's model is similar to Kimber and Hollis. Under these 'typical' link performance functions, good solution methods are well known. We will argue that another stochastic approach is more powerful based on state-dependent queues because it can also handle general service times, multiple servers, and has transient as well as steady-state solutions. It is a true stochastic approach with no approximations. Fig. 1 presents results from many empirical studies for North American roads (Drake et al., 1967; Edie, 1961; Greenshields, 1935; Transportation Research Board, 2000; Underwood, 1961). Obviously congestion may be perceived as a decrease in the mean speed when the vehicular density increases, resulting in the well-known speed-flow-density curves (see e.g. the seminal work by Greenshields (1935), on this).

In particular, we introduce a stochastic version of the SO model in which the costs incurred on each path come from $M/G/c/c$ state-dependent queueing networks. This latter model is a stochastic travel time estimation formula that takes into account these important congestion effects. The $M/G/c/c$ state-dependent queueing models originated with the work of Yuhaski and Smith (1989) for pedestrian traffic flows. This paper formed the foundation of all the subsequent models used in this approach to the travel time flow modeling problem. Following this were the papers of Cheah and Smith (1994) which generalized the process and showed that the state-dependent queue was quasi-reversible and Jain and Smith (1997) which showed how the state-dependent queues could be used for modeling vehicular congestion. In Section 3.1.2, we will describe in detail the elaboration of the $M/G/c/c$ state-dependent queueing model. For a review on the use of queueing models to model traffic flows and congestion, the reader is referred to the paper by van Woensel and Vandaele (2007). Another successful attempt to refine the travel time estimation may be found in the paper by García-Ródenas et al. (2006).

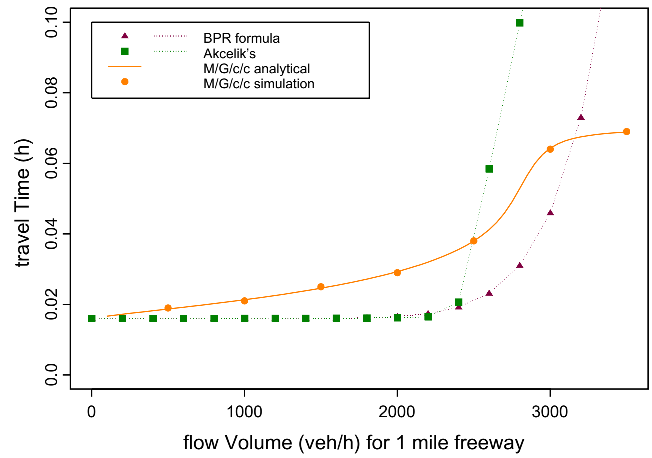

Fig. 2. One-mile vehicular traffic flows.

Fig. 2 shows typical travel time functions (recently used, for instance, by Ghatee and Hashemi (2009) and Pursals and Garzón (2009)) in comparison with the $M/G/c/c$ state-dependent queueing model functions (Jain and Smith, 1997), for a 1-mile long freeway, with free-flow speed 62.5 miles per hour (100 kilometer/hour), and capacity 2400 veh/hour, based on the Highway Capacity Manual (Transportation Research Board, 2000). In addition, Fig. 3 shows how the travel time functions behave as a function of the arrival rate for several single links admitting an $M/G/c/c$ state-dependent queue to model the road traffic. Note that under low traffic, the queueing approach is close to classical and accurate formulas, such as BPR and Akçelik's, as seen in Fig. 2.

Important to note from Figs. 2 and 3 is that the $M/G/c/c$ travel time function is not convex but *S*-shaped, which will produce many local optima. Consequently, the introduction of these stochastic $M/G/c/c$ state-dependent models will make the SO problem computationally more challenging as multiple solutions may be present. The Frank–Wolfe algorithm is a convex combination algorithm (Frank and Wolfe, 1956) that has been often used for determining the equilibrium flows in transportation networks.

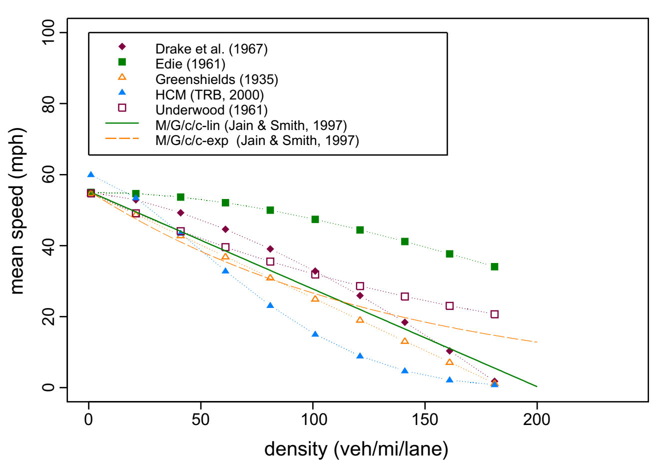

Fig. 1. Empirical distributions for vehicular traffic flows (Drake et al., 1967; Edie, 1961; Greenshields, 1935; Transportation Research Board, 2000; Underwood, 1961) and $M/G/c/c$ state-dependent models (Jain and Smith, 1997).

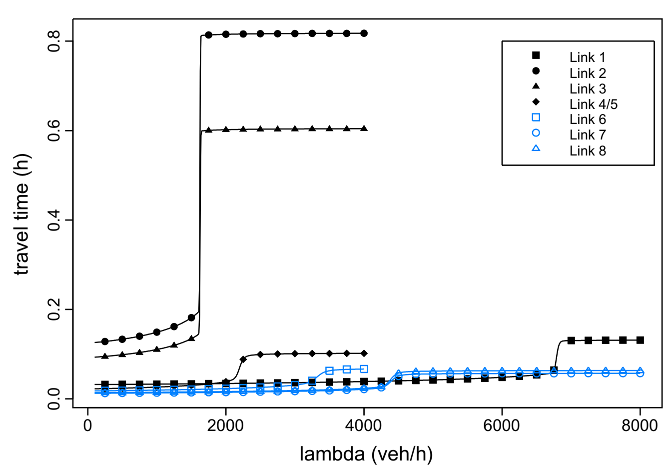

Fig. 3. Travel time under $M/G/c/c$ state-dependent models.