European Journal of Operational Research 201 (2010) 259–266

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

An adjustment scheme for nonlinear pricing problem with two buyers

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ARTICLE INFO

Article history:
Received 21 January 2008
Accepted 23 January 2009
Available online 5 February 2009

Keywords:
Pricing
Buyer–seller game
Limited information
Online computation
Adjustment

ABSTRACT

We examine a contracting problem with asymmetric information in a monopoly pricing setting. Traditionally, the problem is modeled as a one-period Bayesian game, where the incomplete information about the buyers' preferences is handled with some subjective probability distribution. Here we suggest an iterative online method to solve the problem. We show that, when the buyers behave myopically, the seller can learn the optimal tariff by selling the product repeatedly. In a practical modification of the method, the seller offers linear tariffs and adjusts them until optimality is reached. The adjustment can be seen as gradient adjustment, and it can be done with limited information and so that it benefits both the seller and the buyers. Our method uses special features of the problem and it is easily implementable.

1. Introduction

We consider a monopoly pricing problem where the market consists of a seller and buyers with different preferences. The buyers are sorted into two classes, and the demand behavior of each class is specified by a utility function. The seller designs a single price schedule as a function of quantity to maximize his profit, from which the buyers select the quantity they wish to consume. In economics and game theory literature this problem is known as the nonlinear pricing problem. More broadly, such a problem falls into the class of principal-agent games where a principal (here a seller) proposes a contract to an agent (a buyer) whose preferences are the agent's private information. In addition to nonlinear pricing and monopoly pricing [12,14,21,22], other examples of such games are optimal taxation [13], regulation [1], and the design of auctions [15]. In the literature all these games are called adverse selection or mechanism design problems; [8,18] are good textbook presentations on the topic.

An essential feature of all adverse selection problems is incomplete information: the principal does not know the exact values of agents' type parameters, although he knows their probability distributions and the functional forms of the agents' utility functions depending on these parameters. Hence, the problem is solved mathematically as a one-shot Bayesian game.

In nonlinear pricing a practical approach to handle incomplete information in an offline manner[1] was suggested by Spence [21], who noted that the buyers' demand functions can be estimated by offering unit prices to the buyers. Wilson [23,24] took the idea further by formulating the problem so that it could be solved by using the demand data that is estimated from the buyers' responses to linear tariffs; see also Räsänen et al. [17] for one such application in electricity markets. The Wilson's approach may, however, require an extensive data collection that can be rather costly; in the case of Räsänen et al., it took three years to collect reasonable consumer demand data to solve a three quantity, two buyer class pricing problem. In Braden and Oren [6] a Bayesian learning formulation over a finite time horizon was studied in an optimal control fashion to estimate the type for one consumer class. As the authors say, the paper provides more insights than numbers to a rather involved problem containing continuous random variables.

Currently, Internet is taking a vital role as an e-commerce platform. Internet is also used for extensive customer data gathering for pricing services and goods. At the same time, however, customer privacy considerations attached to data collection matter and should be taken into account in the analysis [9]. This fact favors development of efficient online[1] pricing schemes that acquire data incrementally rather than offline pricing methods which usually need large customer data set to be applicable. In papers dealing with dynamic pricing of goods, where in addition to varying demand also inventory considerations may count, various online learning methods have been used to forecast the correct customer behavior and future demand curve [11,16]. Brooks et al. [5] consider adjustment of different pricing schedules, e.g., linear, two-part, nonlinear, etc. tariffs, in nonlinear pricing setting where monopolist offers consumers a new set of articles in each time period. One question they emphasize is that learning customer preferences takes time during which the seller earns less than the optimal profit. In addition to OR literature, the development of

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[1] Using an offline algorithm to solve a problem at hand requires that the whole problem data is available from the beginning. In contrast, an online algorithm can process its data incrementally, without having the entire data available from the start.

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2009.01.037

computational algorithms for games that use limited amount of information about the other agents' preferences, e.g., multiagent learning algorithms and combinatiorial auction algorithms, have recently been under active research in AI literature, too [4,19,20].

In this paper we assume that the seller knows the number of different buyers, but does not have knowledge on their utility functions. Instead we assume that the product is sold repeatedly to myopic buyers. By observing the realized sales the seller plans a better pricing policy for the next period. We first present a discrete step adjustment scheme to solve the problem in an online fashion. Actually we come to this scheme intuitively by requiring that the seller increases the amount to be sold a little bit in every period and that in every such period both the seller and the buyers should gain. It turns out that the resulting method is a steepest ascent method. To avoid too many price changes in every iteration step we then consider a practical modification of the method by making use of linear tariffs. This kind of adjustment problem has not been studied in the literature earlier; see however [7,10] where linear tariff adjustment scheme for one buyer type was studied. This paper shows that there are intuitively appealing computational schemes for solving the problem with several buyer types, too.

The contents of the paper are as follows. In Section 2, we formulate the nonlinear pricing problem and study its optimality conditions. We also study an illustrative example in detail. In Section 3, we present a discrete step heuristic method and discuss its properties. It turns out that the invented method can be considered a discrete step gradient adjustment scheme. In Section 4, we define and analyze a modified method based on the use of linear tariffs. In Section 5, we simulate numerically the performance of our method, and finally in Section 6, we offer further considerations to the issue.

2. Model

A firm, the seller denoted by S, produces a product x, $x \geqslant 0$, to a population of buyers. The seller differentiates the buyers by offering them different quantities of the product. We assume that there are just two types of buyers in the population: a low buyer and a high buyer denoted from now on by L and H, respectively. The buyers' utilities are quasi-linear,

$$U_i(x,t) = V_i(x) - t, \quad i = L, H, \tag{1}$$

where t is the price of the product and $V_i(x)$ is buyer i's gross surplus of consuming quantity x. The utilities are scaled so that $V_i(0) = 0, i = L, H$. The gross surplus $V_i(x)$ is assumed to be twice continuously differentiable, increasing and strictly concave, i.e., $V_i'(x) \geqslant 0$, $V_i''(x) < 0$, when $x \geqslant 0$.

The seller offers the buyers two types of quantity-price bundles, (x_L, t_L) and (x_H, t_H), and gets a total profit

$$\pi(x_L, x_H, t_L, t_H) = p_L(t_L - c(x_L)) + p_H(t_H - c(x_H)), \tag{2}$$

where p_i is the relative number of buyers i in the population, and $c(x)$ is the seller's cost of producing quantity x. Without loss of generality, we assume that there is only one L buyer and one H buyer with weights p_L and p_H, respectively. Furthermore, we assume that the production cost is of the form $c(x) = cx$, where $c \geqslant 0$ is a constant. We note, however, that the production cost could be convex as well and this would result only in minor changes in the rest of the paper.

In the market the buyers self-select the bundle they wish to consume. In maximizing his profit, the seller therefore faces two kinds of constraints: *individual rationality* (IR) constraints

$$U_i(x_i, t_i) = V_i(x_i) - t_i \geqslant U_i(0,0) = 0, \quad i = L, H, \tag{3}$$

and *incentive compatibility* (IC) constraints

$$U_i(x_i, t_i) = V_i(x_i) - t_i \geqslant V_i(x_j) - t_j = U_i(x_j, t_j), \quad j \neq i. \tag{4}$$

The IR constraints state that a buyer should get positive utility when choosing the bundle intended for him. The IC constraints let the buyers *self-select* the bundle for them; the buyers prefer their own bundle the most. Now, the seller's problem is maximization of $\pi(x_L, x_H, t_L, t_H)$ with respect to the constraint equations (3) and (4).

2.1. Necessary and sufficient optimality conditions

We derive first-order conditions to the problem by making a common assumption used in literature, which states that the buyers' utility functions can be sorted.

Assumption 1. $V_H'(x) > V_L'(x), \quad \forall x \geqslant 0.$

This assumption is called the *single-crossing property* and it has two major implications. First, the optimal quantities are increasing in buyer type, $x_H^* \geqslant x_L^*$, where from now on * refers to the optimality. Second, the optimal prices are

$$t_L^* = V_L(x_L^*), \tag{5}$$
$$t_H^* = t_L^* + V_H(x_H^*) - V_H(x_L^*). \tag{6}$$

These results are derived in Spence [22]. Using these results, we can simplify the seller's problem to

$$\begin{aligned} \max_{x_L, x_H, t_L, t_H} \quad & \pi(x_L, x_H, t_L, t_H) = p_L(t_L - cx_L) + p_H(t_H - cx_H) \\ \text{s.t.} \quad & t_L = V_L(x_L), \\ & t_H = t_L + V_H(x_H) - V_H(x_L), \\ & x_H \geqslant x_L \geqslant 0. \end{aligned} \tag{7}$$

Assumption 2. There is $x_i^E > 0$ so that $V_i'(x_i^E) = c$, $i = L, H$.

This assumption rules out the possibility that selling nothing to both buyers is optimal for the problem. If buyer i was alone in the market, he would be served with the amount x_i^E, which is called the *first-best solution*. In this case, when the cost is linear and V_i is strictly concave, this amount is unique.

Let us define $f_L(x) = p_L(V_L(x) - cx) - p_H(V_H(x) - V_L(x))$ and $f_H(x) = p_H(V_H(x) - cx)$. Then substituting the equality constraints in (7) into the objective function, we get $\pi(x_L, x_H, t_L, t_H) = f_L(x_L) + f_H(x_H)$. Hence, forgetting the constraints $x_H \geqslant x_L \geqslant 0$ for a while, we get the necessary conditions of (7) for a solution $0 < x_L^* \leqslant x_H^*$,

$$f_H'(x_H^*) = p_H(V_H'(x_H^*) - c) = 0, \tag{8}$$
$$f_L'(x_L^*) = p_L(V_L'(x_L^*) - c) - p_H(V_H'(x_L^*) - V_L'(x_L^*)) = 0. \tag{9}$$

Assumptions 1 and 2 imply that $0 < x_L^E < x_H^E < \infty$, and that $f_L'(x) < 0$, for all $x \geqslant x_L^E$. Thus for a solution of (9) we have $0 \leqslant x_L^* < x_L^E$. By (8), $x_H^* = x_H^E$, hence it also holds that $x_L^* < x_H^*$. But (9) may not have solution at all, since $f_L'(x)$ can be strictly negative for all $x \in [0, x_L^E]$. Thus, the problem solution is either to serve both buyers or to exclude the low type and serve only the high type. Which case will happen depends on the buyers' utilities and weights p_L and p_H. The latter case will happen if p_L is small, or if the low type values the product considerably less than the high type. If this is the case, the solution is given by $x_H^* = x_H^E$, $t_H^* = V_H(x_H^*)$, and $x_L^* = t_L^* = 0$. In this paper, we shall assume that it is optimal to serve both buyers. Therefore, we make the following assumption.