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A deterministic tabu search algorithm for the fleet size and mix vehicle routing problem

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Abstract

The fleet size and mix vehicle routing problem consists of defining the type, the number of vehicles of each type, as well as the order in which to serve the customers with each vehicle when a company has to distribute goods to a set of customers geographically spread, with the objective of minimizing the total costs. In this paper, a heuristic algorithm based on tabu search is proposed and tested on several benchmark instances. The computational results show that the proposed algorithm produces high quality results within a reasonable computing time. Some new best solutions are reported for a set of test problems used in the literature.

Keywords: Tabu search; Heuristics; Vehicle routing; Heterogeneous fleet; Fleet mix; Logistics

1. Introduction

The fleet size and mix vehicle routing problem (FSMVRP) can be described mathematically as follows. Let $G = (V, E)$ be an undirected connected graph with a vertex set $V = \{0, 1, \ldots, n\}$ and an edge set $E = \{(i,j)\colon i,j \in V\}$. Vertex 0 represents the depot and each other vertex $i \in V\backslash\{0\}$ is a customer with a non-negative demand q_i. A distance d_{ij} ($d_{ii} = 0,\ \forall i \in V$) is associated to each edge $(i,j) \in E$. There is a fleet of T different types of vehicles located at the depot, and the number of vehicles of each type is considered unlimited. A capacity Q_k, a fixed cost F_k, which is incurred by simply using a vehicle, and a variable cost v_k are associated to each type of vehicle k ($k = 1, \ldots, T$). We assume that $Q_1 < Q_2, \cdots, < Q_T$ and $F_1 < F_2, \cdots, < F_T$. The travelling cost of each edge $(i,j) \in E$ by a vehicle of type k is $c_{ij} = v_k d_{ij}$. The FSMVRP consists of defining a set of routes and the vehicles assigned to them so that the following constraints are taken into account: (i) satisfy customers' demand; (ii) visit each customer exactly once; (iii) a vehicle route starts and finishes at the depot, and the capacity of the vehicle is not exceeded. The objective of the FSMVRP is to minimize the sum of fixed and variable costs of all the routes subject to the previous constraints.

A particular case of FSMVRP happens when $T = 1$, i.e., when the fleet is homogeneous, which is the classical vehicle routing problem (VRP), should the fixed costs not be taken into consideration. Therefore, we can conclude that, like the VRP (this is proven, for example, in Lenstra and Rinnoy Kan (1981)), the FSMVRP is a *NP-hard* combinatorial problem.

The FSMVRP has been studied under several other designations like vehicle fleet composition (Etezadi and Beasley, 1983; Salhi and Rand, 1993), mix fleet vehicle routing problem (Wassan and Osman, 2002) and heterogeneous fleet vehicle routing problem (Gendreau et al., 1999; Yaman, 2006; Choi and Tcha, 2007). On the other hand, Taillard (1999), Tarantilis et al. (2004) and Li et al. (2007) studied a variant of the FSMVRP with a fixed number of vehicles for each type. Another variant of this problem, the FSMVRP with time windows has been studied by researchers such as Liu and Shen (1999) and Dullaert et al. (2002).

In practice, many companies face distribution problems similar to the FSMVRP or, more commonly, with some

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additional constraints. The need for different types of vehicles is determined by the characteristics of the customers. Usually, larger vehicles are more appropriate for serving customers who require large orders and the smaller vehicles are more adequate to deliver small quantities or serve customers that have access restrictions. The FSMVRP may be present either when making strategic decisions or in the operational ones. The strategic (or tactical) decision is when the fleet is chosen and the company must decide which type of vehicles to buy and how many of each type, i.e., the company wishes to define the best composition of the fleet. In this case, the most relevant costs to be included in the objective function are the fixed costs of the vehicles. The operational decisions appear in the daily planning, once the fleet of vehicles has been defined, since the total costs will depend on the number and type of vehicles effectively used for the deliveries, as well as on the routes in which they are used. Besides, when the owned vehicles are not enough, a decision has to be made regarding which types of vehicles should be hired and how many of each. For the daily planning the most important costs to be considered in the objective function are the variable costs, although the fixed costs may also be included, especially in relation to the hired vehicles, depending on the kind of contract.

The VRP has received a lot of attention and there is a vast amount of papers and books about it. As a consequence of this, there has been an enormous progress in its resolution. A survey of the existing resolution methods may be found, for example, in Laporte (1992), Golden et al. (1998) and Laporte et al. (2000), and an extensive bibliography is presented in Laporte and Osman (1995). Contrarily, the FSMVRP, in spite of its practical importance, has been much less studied. Among these, deserve mention the works of Semet and Taillard (1993), Rochat and Semet (1994) and Brandão and Mercer (1997), which are some examples of real-life problems that include the FSMVRP with several additional constraints. In the case of Brandão and Mercer (1997), the company owned a fleet of two types of vehicles and, whenever necessary, they could complement their fleet with hired vehicles. Some of the additional constraints were the following: some customers could only be served by one vehicle type, time windows for serving the customers, each vehicle could perform more than one route per day (a more detailed description can also be found in Brandão (1994)). Below, we will briefly review some of the literature on FSMVRP resolution methods.

As far as we know, no exact methods have been developed for the FSMVRP. However, three papers dealing with the determination of lower bounds deserve mention: Golden et al. (1984), Yaman (2006) and, more recently and with better results, Choi and Tcha (2007). In the latter, the authors were able to find lower bounds, with a gap, in relation to the best known solution, ranging from 0.17% to 4.32% for a set of 36 problems from the literature, containing between 20 and 100 customers. The average gap was only 1.71%.

Considering the similarities between the VRP and the FSMVRP, no wonder that most of the methods developed for solving the latter are adaptations of methods already widely used for the former. Golden et al. (1984) have created several heuristics based on the savings method of Clarke and Wright (1964), as well as on the giant tour algorithm of Gillett and Miller (1974). Gheysens et al. (1986) adapted the generalised assignment heuristic of Fisher and Jaikumar (1981). Desrochers and Verhoog (1991) have adapted a matching based savings heuristic initially proposed by themselves for the VRP. The heuristic of Salhi and Rand (1993) is also based on a previous one developed by themselves for the VRP. This heuristic was later improved by Osman and Salhi (1996). All these methods, more or less sophisticated, can be classified as classical heuristics. The most recent heuristic of this type is due to Renaud and Boctor (2002) and is, among them, the one that produces better quality solutions, but also the one that requires more computing time. This heuristic starts by generating a large set of routes using different procedures and afterwards chooses those that satisfy the constraints of the problem at the lowest cost using an exact, but polynomial, set partitioning algorithm. Among those procedures of producing the initial set of routes is the petal method used by several authors, including Renaud et al. (1996), for the VRP.

More recently, meta-heuristics have been proposed for solving the FSMVRP: Osman and Salhi (1996), Gendreau et al. (1999) and Wassan and Osman (2002), which will be designated from now on as OS, GLMT and WO, respectively. All these algorithms are based on tabu search. The OS algorithm takes an initial solution produced by the heuristic of Salhi and Rand (1993), which is then improved by a tabu search method with short term memory and with moves defined by a 1-interchange mechanism. The GLMT algorithm is rather complex and requires the use of GENIUS, developed by Gendreau et al. (1992) for the travelling salesman problem (TSP), as well as an adaptive memory procedure developed by Rochat and Taillard (1995). The WO algorithm comprises several variants obtained from the selection of different neighbourhood mechanisms, tabu restrictions and tabu tenure schemes.

Very recently, Choi and Tcha (2007) developed a mathematically based resolution method for the FSMVRP. In this method, the FSMVRP is formulated as a set covering problem as it is done for VRP with time windows. Then, its linear programming relaxation is solved by the column generation technique. This method could not solve exactly any of the examples tested, but it generated good lower bounds and upper bounds and, therefore, can be used as a good heuristic method.

The structure of the remainder of this paper is as follows. Section 2 describes the methods used to obtain initial feasible solutions in our algorithm. Section 3 describes the tabu search algorithm (TSA) for solving the FSMVRP, presenting its main features and a flow chart that shows how the different components interact. In Section 4, the proposed parametric setting is discussed, and are given