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Suspension model for blood flow through catheterized curved artery with time-variant overlapping stenosis

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ABSTRACT

This paper is concerned with the analysis of a particle-fluid suspension model for the axi-symmetric flow of blood through curved coaxial tubes where the outer tube with mild overlapping stenosis while the inner tube is uniform rigid representing catheter. The governing equations written in rectangular toroidal coordinates and the problem is formulated using a perturbation expansion in terms of a variant of curvature parameter to obtain explicit forms for the axial velocities of fluid and particulate phases, the stream function, the resistance impedance, pressure drop and the wall shear stress distribution also the results were studied for various values of the physical parameters, such as the curvature parameter ε, the radius of catheter σ, the volume fraction density of the particles C, the taper angle ϕ and the maximum height of stenosis δ^*. The obtained results show that there is a significant deference between curvature and non-curvature annulus flows through catheterized stenosed arteries. This study provides a scope for estimating the influence of the problem parameters on different flow characteristics and to ascertain which of the parameters has the most dominating role.

1. Introduction

The detection of atherosclerotic lesions at an early stage of the disease, which means only minimal narrowing of the arterial lumen, is of high clinical interest. These vessels pose grave health risks and are a major cause of mortality and morbidity in the industrialized world. Narrowing of a blood vessel, or stenosis, is defined as a partial occlusion of the blood vessels due to the accumulation of cholesterol, fats and the abnormal growth of tissue [1]. Stenosis is one of the most frequent anomaly in blood circulation, and once the construction is formed, the blood flow is significantly altered and fluid dynamical factors play important roles as the stenosis continues to enlarge leading to the development of cardiovascular diseases such as heart attack and stroke. The physiological importance of studies pertaining to the variation of the resistance to flow and the wall shear stress with the axial distance and the depth of the stenosis were discussed by many authors [2–5]. The use of catheters is of immense importance and has become a standard tool for diagnosis and treatment in modern medicine. A catheter is made of polyester based thermoplastic polyurethane, medical grade polyvinyl chloride, etc. Catheters are also being used in diagnostic techniques (e.g. X-ray angiography, intravascular ultrasound). For the purpose of flexible PVC materials containing added plasticizers are used in catheters which enable them to move through the branches or curved paths of the circulatory system. In routine clinical studies and animal experiments, the measurement of arterial blood pressure/pressure gradient and flow velocity/flow rate is usually achieved by the use of an appropriate catheter-tool device (such as a catheter-transducer system or a catheter tip flow meter) in the desired part of the arterial network.

In balloon Angioplasty, inflatable balloon catheters is widely used to clear short occlusions due to the atherosclerosis. Also, the catheter is carefully guided to the location at which stenosis occurs in the coronary arteries and the balloon is then inflated to fracture the fatty deposits and widen the narrowed portion. An inserted catheter in an artery will increase the impedance and will modify the pressure distribution and alter the flow field. Consequently, the pressure recorded by the attached transducer to the catheter will differ from that of an uncatheterized artery. An estimate of catheter induced error to have an accurate pressure reading is therefore essentially required.

Recently, there has been a considerable increase in the use of catheters of various sizes. It has been shown that, by reducing the

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obstruction through balloon angioplasty, the mean translational pressure drop Δp is reduced (Anderson et al., 1986) [2] and the coronary blood flow as well as the coronary flow reserve is increased (Wilson et al., 1988) [3]. The magnitude of mean translational pressure drops Δp is often used by clinicians to gage the severity of the lesion.

Blood is a marvelous fluid which nurtures life. For over a couple of centuries, the theoretical and experimental studies of blood flow through the circulatory system of living mammals, has been the subject of scientific research. The experimental observations and the theoretical analysis of blood flow are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in animal or human physiology. Mathematical modeling of blood flow has been subject to modifications in order to account for the new evidences uncovered through the improved initial experimental observations [6]. Moreover, the continuum theory of mixtures (particulate suspension) is applicable to the hydrodynamics of biological systems, because it provides an improved understanding of diverse subjects such as diffusion of proteins, the rheology of blood, swimming of microorganisms and particle deposition in the respiratory tract. Being a suspension of corpuscles, at low shear rate, blood behaves like a non-Newtonian fluid.

Also, it is well known from the investigations of Haynes [4] and Cokelet et al. [5] that blood can no longer be treated as single-phase homogeneous viscous fluid while flowing through narrow arteries (of diameter $\leq$ 1000 μm). Srivastava and Srivastava (1983) [6] observed that the individuality of red cells (of diameter 8 μm) is important, even in such large vessels of diameter up to 100 cell diameter. Thus, for an accurate description of blood flow requires the consideration of erythrocytes (red cells) as discrete particles in small blood vessels, i.e., a two-phase fluid, where, the particulate phase is the red blood cells and the fluid phase is the plasma [7,8].

The study of flow in a curved tube with constriction has possible applications to blood flow in stenotic arteries-constricted arteries. As a matter of fact, Identification of the local vascular geometry and, consequently, a detailed profile of intravascular hemodynamics, may be valuable in the management of coronary patients. Previous research has shown that arterial bends and bifurcations have a significant impact on the hemodynamic factors. Extensive clinical and computational studies have been performed to investigate fluid dynamics in bifurcations and curved arteries. Liu et al. [9] studied the effects of stenosis in curved arteries. They found complex, disturbed flow distal to plaque that may contribute to the development of current lesions or genesis of new plaques. Moreover, the geometry of most physiological conduits and glandular ducts is curved. Model of micro wrinkles on human skin also requires a curved geometry. The geometry of the airways and the arterial network produces swirling flows, similar to the flows found in curved or twisted pipes [10,11].

The important contributions of recent years to the topic are referenced in the literature [12–16]. Many of researches about arteriosclerotic development indicate that the studies are mainly concerned with the single symmetric and non-symmetric stenoses while the stenoses may develop in series (multiple stenoses) or may be of irregular shapes or overlapping or of composite in nature. Some studies considered an overlapping stenosis in the blood vessel segment. Chakravarty and Mandal [17] noted that the problem becomes more acute in the presence of an overlapping stenosis in the artery instead of a mild one. The effect of vessel tapering is another important factor that was considered. Chakravarty and Mandal [18] formulated the problem on tapered blood vessel segment having an overlapping stenosis. Ismail et al. [19] studied the power-law model of blood flow through an overlapping stenosed artery where an improved shape of the time-variant stenosis in the tapered arterial lumen is given and the vascular wall deformability is taken to be elastic (moving wall). Recently, Mandal [20] investigated the unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis. Mekheimer et al. [21] analyzed the effect of the induced magnetic field on blood flow through an anisotropically tapered elastic arteries with overlapping stenosis in an annulus. Mekheimer et al. [22] investigated the influence of heat and chemical reactions on micropolar fluid through anisotropically tapered elastic artery with overlapping stenosis. Mekheimer and El Kot [23] presented a mathematical modeling of unsteady flow of a Sisko fluid through an anisotropically tapered elastic arteries with time-variant overlapping stenosis. More investigations for the blood flow through a stenosed arteries can be found in [24–33].

With the above discussion in mind, the goal of this investigation to study the effect of catheterization on various physiologically important flow characteristics (i.e. pressure drop, impedance and wall shear stress) as well as on the flow patterns in a curved artery with time-variant overlapping stenosis through a mathematical model. The problem is first modeled and then the non-dimensional governing equations for blood (particulate suspension model, i.e., a suspension of erythrocytes in plasma) are formulated in rectangular toroidal coordinates. The non-dimensional governing equations in the case of mild stenosis and the corresponding boundary conditions are prescribed then solved analytically for the axial velocity. The results are used to obtain the estimates of increased pressure drop across a coronary artery stenosis during catheterization. In addition, many interesting fluid mechanical phenomena, i.e. streamlines due to the combined action of stenosis and curvature, are brought out in this paper. Finally, the main finding of the results is summarized as concluding remarks. The insight thus gained may be useful in relating the results of laboratory investigations of endothelial behavior under different shear stress conditions to clinical observations and will provide a foundation for future studies of local shear stress, plaque behavior, and ultimately, clinical cardiovascular events.

2. Mathematical formulation

2.1. Flow geometry

Consider the particle-fluid suspension model of blood flowing through a curved artery of finite length L with overlapping stenosis. The mathematical formulation model of the curved artery as a rigid circular tube of radius R_o coiled in the form of a circle of radius b and the catheter as a coaxial rigid tube with radius σR_o with $\sigma < 1$. It is assumed that the stenosis has developed in an axi-symmetric manner due to some abnormal growth over a length $3L_o/2$ of the artery and the geometry of arterial wall with time-variant overlapping stenosis for different taper angles (see Fig. 1) is written mathematically as [19].

$$
\begin{aligned}
R(z,t) &= \left[(mz+R_o) - \frac{\delta\cos\phi}{L_o}(z-d)\left\{11 - \frac{94}{3L_o}(z-d) + \frac{32}{L_o^2}(z-d)^2 - \frac{32}{3L_o^3}(z-d)^3\right\}\right]\Omega(t) \quad d \le z \le d + \frac{3L_o}{2} \\
&= (mz+R_o)\Omega(t) \qquad \text{otherwise},
\end{aligned}
\tag{1}
$$