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European Journal of Operational Research 187 (2008) 120–137

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

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Stochastics and Statistics

Mean–variance portfolio and contribution selection in stochastic pension funding

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Received 20 July 2006; accepted 1 March 2007
Available online 4 April 2007

Abstract

In this paper we study the problem of simultaneous minimization of risks, and maximization of the terminal value of expected funds assets in a stochastic defined benefit aggregated pension plan. The risks considered are the solvency risk, measured as the variance of the terminal fund's level, and the contribution risk, in the form of a running cost associated to deviations from the evolution of the stochastic normal cost. The problem is formulated as a bi-objective stochastic problem of mean–variance and it is solved with dynamic programming techniques. We find the efficient frontier and we show that the optimal portfolio depends linearly on the supplementary cost of the fund, plus an additional term due to the random evolution of benefits.

Keywords: Finance; Pension funding; Portfolio theory; Stochastic control; Mean–variance

1. Introduction

The optimal management of dynamic pension plans is an interesting problem due to the importance that pension funds have currently in financial markets, as well as their fundamental role in assuring the future wealth of participants in their retirement period.

Pension funds can be classified into the following two main categories: defined benefit (DB) pension plans and defined contribution (DC) pension plans. In a DB plan benefits are fixed in advance by the sponsor and contributions are designed to amortize the fund according to a previously chosen actuarial scheme. Future benefits due to participants are thus a liability for the sponsor, who bears the financial risk. Of course, this risk is increased with the formation of a risky portfolio that, however, offers higher expected returns, with the possibility then of reducing the amortization quote. It is the concern of the sponsor to drive the dynamic evolution of the fund having into account the trade-off between risk and contribution. In a DC plan contributions are fixed but benefits depend on the returns of the fund portfolio, so that the participants bear the risk.

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doi:10.1016/j.ejor.2007.03.002

Beginning with the paper of Haberman and Sung (1994), DB plans have been usually modelled as linear-quadratic optimal control problems.[1] This is due to the fact that the dynamics of the fund is postulated linear, as in Merton's model, and that it is generally accepted that managers' objectives should be related with the minimization of solvency risk and contribution risk. These risk concepts are defined as quadratic deviations of fund wealth and amortization rates with respect to liabilities and normal cost, respectively. In an environment where liabilities are random, the risks so formulated do not correspond to the variance, which is by far the most common measure of risk. The aim of this paper is to study the optimal management of DB plans when the solvency risk is identified with the variance of the unfunded actuarial liability. To this end, the problem is settled in the familiar mean–variance framework, translating the static model of Markowitz to the continuous-time setting of a DB plan that evolves with time.

Markowitz (1952) designed the mean–variance model to compare securities and portfolios based in a trade-off between their expected return and risk, measured as the variance of the return. From the point of view of optimization, the problem of portfolio selection is a multiobjective programming problem where it is desired to attain the highest possible expected return with the lowest possible variance. Since these objectives are in general mutually incompatible, the best can be done is to select portfolios where it is not possible to increase return without increasing risk, and reciprocally, where it is not possible to decrease risk without decreasing return. The set of pairs (return, variance) enjoying theses properties are called the Pareto frontier or efficient points set, and the associated portfolios are called efficient.

It has been several attempts in the literature to translate the mean–variance methodology from the static case to the dynamic setting. The most successful and fundamental is of course the one initiated by Merton. It is worth noting, however, that Merton's model does not exactly fit the structure of the mean variance approach. It has been recently, in the papers by Zhou and Li (2000) and Li and Ng (2000) that the methodology has been more faithfully carried out to the dynamic setting, in continuous and in discrete time, respectively. In our paper we follow the formulation of Zhou and Li (2000) but with some modifications due to the specificities of a DB plan, as the inclusion of the supplementary cost as a control variable in addition to the quantities invested in the risky assets. This point is explained in Section 3 below. Moreover we use the Hamilton–Jacobi–Bellmam approach instead of the maximum principle.

Problems of mean–variance type have been recently considered in pension plans from a static point of view in Colombo and Haberman (2005) and in Huang and Cairns (2005). A dynamic model for asset and liability management under the mean–variance criteria has been studied in Chiu and Li (2006). The framework provided by these authors, although general, cannot be applied directly to a DB plan since several of the constitutive elements of the pension plan, as the amortization rate, normal cost, benefits and the technical actuarial rate, are not contemplated in the model. More fundamentally, a DB plan is identified by two different elements of control: investment decisions in the portfolio and amortization rate. The latter is absent in the framework provided by Chiu and Li (2006). The existence of an additional control variable requires a modification in the objective functional, introducing a running cost associated to the size of the amortization rate, more concretely, associated to quadratic deviations with respect to the stochastic normal cost. Thus our problem combines terminal payoffs due to the final levels of expected surplus/debt and of the variance of fund wealth (the *stock* variable) as well as an integral term or running cost that takes care of the contribution risk (the amortization rate is a *flow* variable).

Our paper follows Josa-Fombellida and Rincón-Zapatero (2004), where the benefits of the DB plan are stochastic, modelled by a geometric Brownian motion. Note that benefits is a non-tradable asset, hence the market is incomplete and, furthermore, we also consider the existence of correlation between the sources of uncertainty in the benefits and in the asset returns.

The paper is organized as follows. Section 2 defines the elements of the pension scheme and describes the financial market where the fund operates. Section 3 is devoted to formulate the management of the DB plan in a mean–variance framework, with the simultaneous objectives of minimizing the expected unfunded actuarial liability, as well as its variance at the final time, and to minimize the contribution rate risk over the planning

[1] There is a growing amount of papers devoted to the optimal management of DB pension plans (see e.g. Haberman and Sung, 1994; Chang, 1999; Cairns, 2000; Haberman et al., 2000; Taylor, 2002; Chang et al., 2002; Josa-Fombellida and Rincón-Zapatero, 2001, 2004, 2006a,b).