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Discrete time modeling of mean-reverting stochastic processes for real option valuation

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Abstract

In this paper the recombining binomial lattice approach for modeling real options and valuing managerial flexibility is generalized to address a common issue in many practical applications, underlying stochastic processes that are mean-reverting. Binomial lattices were first introduced to approximate stochastic processes for valuation of financial options, and they provide a convenient framework for numerical analysis. Unfortunately, the standard approach to constructing binomial lattices can result in invalid probabilities of up and down moves in the lattice when a mean-reverting stochastic process is to be approximated. There have been several alternative methods introduced for modeling mean-reverting processes, including simulation-based approaches and trinomial trees, however they unfortunately complicate the numerical analysis of valuation problems. The approach developed in this paper utilizes a more general binomial approximation methodology from the existing literature to model simple homoskedastic mean-reverting stochastic processes as recombining lattices. This approach is then extended to model dual correlated one-factor mean-reverting processes. These models facilitate the evaluation of options with early-exercise characteristics, as well as multiple concurrent options.

The models we develop in this paper are tested by implementing the lattice in binomial decision tree format and applying to a real application by solving for the value of an oil and gas switching option which requires a binomial model of two correlated one-factor commodity price models. For cases where the number of discrete time periods becomes too large to be solved using common decision tree software, we describe how recursive dynamic programming algorithms can be developed to generate solutions.

Keywords: Decision analysis; Stochastic processes; Finance; OR in energy

1. Introduction

Discounted cash flow methods (DCF) are commonly used in practice for the valuation of projects and for decision-making regarding investments in real assets. The seminal work of Black and Scholes (1973) and Merton (1973) in the area of financial option valuation led to the application of option pricing methods in valuing real investments under uncertainty, by recognizing the analogy between financial options and project decisions that can be made after some uncertainties are resolved. This

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approach has the advantage of including the value of managerial flexibility, which is frequently not captured by the traditional DCF approach.

The options derived from managerial flexibility are commonly called ‘‘real options’’ to reflect their association with real assets rather than with financial assets. Despite its theoretical appeal, however, the practical use of real option valuation techniques in industry has been limited by the mathematical complexity of these techniques and the resulting lack of intuition associated with the solution process, or the restrictive assumptions required to obtain analytical solutions.

The mathematical complexity associated with option theory stems from the fact that the general problem requires a probabilistic solution to a firm’s optimal investment decision policy, not only at present but also at all instances in time up to the maturity of its options. To solve this problem of dynamic optimization, the evolution of uncertainty in the value of the real asset over time is first modeled as a stochastic process. Then the value of the firm’s optimal policy over time is obtained as the solution to a stochastic differential equation with appropriate boundary conditions to reflect the initial conditions and terminal payoff characteristics. Recursive dynamic programming may be used to obtain closed-form mathematical solutions for certain types of stochastic processes and for specific exercise characteristics of options.

To provide a transparent, computationally efficient model of the valuation problem, a discrete approximation of the underlying stochastic process can be developed. The first example of this approach was a binomial lattice model that converges weakly to a lognormal diffusion of stock prices known as a Geometric Brownian motion or GBM, developed by Cox et al. (1979). A binomial lattice may be viewed as a probability tree with binary chance branches, with the unique feature that the outcome resulting from moving up and then down in value is the same as the outcome from moving down and then up. There will be the same number of different outcomes in any period for lattice or a tree, though some of the outcomes will be recurring in the binomial tree.

The binomial model can be used to accurately approximate solutions from the Black–Scholes–Merton continuous-time option valuation model. Moreover, this approach can also be used to solve for the value of early-exercise American options, whereas the Black–Scholes–Merton model can only value European options.

However, the assumption of a lognormal geometric Brownian diffusion as a model of the underlying stochastic process may not be valid for many real option valuation problems, such as projects with cash flows that depend on mean-reverting commodity prices. If commodity prices follow a mean-reverting process, they will tend to drift toward a long-term mean price level. There are compelling economic arguments supporting the concept that commodity prices should be mean-reverting. Relatively high commodity prices dampen demand, encourage development of alternative products, and stimulate additional investments to increase the production of the commodity, all of which drive future prices down. Conversely, relatively low prices reduce investments in new sources of production and stimulate demand for the commodity, both conditions which result in higher future prices.

Empirical studies of historical data have found that mean-reverting models accurately capture the evolution of commodity prices (e.g., Schwartz, 1997). Bessembinder et al. (1995) find that a forward-looking analysis of the commodities futures data implies mean reversion as well. As noted by Schwartz (1998), Laughton and Jacoby (1993), and others, if commodity prices are indeed mean-reverting, then a lognormal geometric Brownian diffusion model can significantly overestimate uncertainty in the cash flows from a project, and result in overstated option values.

Discrete-time modeling of mean-reverting stochastic processes has proven problematic, however. Methods employing Monte Carlo simulation and discrete trinomial trees have been the two primary proposed approaches. Monte Carlo methods such as the approach of Longstaff and Schwartz (2001) are able to accommodate general types of stochastic processes, and can be also used to value early-exercise options, a shortcoming of traditional simulation-based methods. However, a significant drawback of this approach is that it is computationally intensive, and particularly so for problems with multiple concurrent options.

Hull (1999) and others have provided examples of discrete tri- and multi-nomial trees for valuing options in a similar manner to the binomial approach, but with the ability to model more general types of stochastic processes, due to the additional degrees of freedom. Unfortunately, these types of trees can be very difficult to implement, since they are path-dependent and branching probabilities