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Joint economic design of EWMA control charts for mean and variance

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Abstract

Control charts with exponentially weighted moving average (EWMA) statistics (mean and variance) are used to jointly monitor the mean and variance of a process. An EWMA cost minimization model is presented to design the joint control scheme based on pure economic or both economic and statistical performance criteria. The pure economic model is extended to the economic-statistical design by adding constraints associated with in-control and out-of-control average run lengths. The quality related production costs are calculated using Taguchi's quadratic loss function. The optimal values of smoothing constants, sampling interval, sample size, and control chart limits are determined by using a numerical search method. The average run length of the control scheme is computed by using the Markov chain approach. Computational study indicates that optimal sample sizes decrease as the magnitudes of shifts in mean and/or variance increase, and higher values of quality loss coefficient lead to shorter sampling intervals. The sensitivity analysis results regarding the effects of various inputs on the chart parameters provide useful guidelines for designing an EWMA-based process control scheme when there exists an assignable cause generating concurrent changes in process mean and variance.

Keywords: Quality control; Average run length; Economic design; EWMA control chart; Markov chain

1. Introduction

Control charts are used for monitoring the level of variation in a production process over time with the objective of reacting quickly to harmful deviations from the normal operating conditions (as well as indicating the effects of process changes and improvements).[1] To implement control charts in practice, control limits, sample size, and sampling frequency must be specified. Once these design parameters are determined, the processes can be controlled against assignable (special) causes leading to undesirable process output. The sample

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[1] Control charts are also used to monitor the impact of process improvements. Its design parameters can then be readjusted to reflect their effects, and then the process remonitored for out-of-control conditions associated with special causes.

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statistics computed based on random samples taken from the process are compared against control limits, and a decision is made regarding whether the process is currently in-control or out-of-control. False alarms occur when an in-control process is erroneously classified as out-of-control (type 1 error) and an assignable cause is searched. There also exists the risk of concluding that the process is in-control based on the sample test statistic although the process is actually out-of-control (type 2 error).

Various types of quality control charts have been proposed to monitor the process mean and/or variability. Although the Shewhart X-bar chart is the most common chart applied in practice to monitor the process mean, in the last 20 years exponentially weighted moving average (EWMA) charts have been offered as a suitable alternative to the X-bar chart for detecting small to moderate shifts in the process mean. Similarly, various alternative control charts exist for monitoring process variation (Acosta-Mejia et al., 1999). Generally control charts for mean and dispersion are used simultaneously since, due to a special cause, one of these two parameters may deviate from its in-control value while the other parameter remains unchanged. Sometimes a single assignable cause may result in changes in both process mean and variation. For example, in integrated circuit manufacturing, the solder paste is printed onto the printed circuit board (PCB) before the mounting of circuit components. The thickness of the solder paste influences the solderability of circuit components on the board. When the process goes out of control, the thickness of the paste is off-target and at the same time the process variance is large since the solder paste thickness is not uniformly distributed over the board (Gan et al., 2004). Thus, process mean and variance are simultaneously affected by the same special cause in this manufacturing setting.

The statistical performance of EWMA based control charts for monitoring the process mean and variation jointly have been studied in several papers (e.g. Morais and Pacheco, 2000; Reynolds and Stoumbos, 2001; Knoth and Schmid, 2002; Reynolds and Stoumbos, 2004). The *statistical design* of control charts takes into account the in-control and out-of-control average run lengths resulting from the sample size and the control limits chosen by the user. Average run length (ARL) is a measure of the expected number of consecutive samples taken until the sample statistic falls outside the control limits. Since ARL is a function of the prevailing process mean and standard deviation, its value depends on whether the process is in-control or out-of-control. When multiple charts are used jointly for monitoring the process, the investigation for an assignable cause is initiated when at least one of the charts triggers an out-of-control signal. Hence, not the ARLs of the individual charts but the joint ARL of the overall control scheme is the relevant performance measure when multiple charts are used simultaneously. Since the statistical design of control charts does not explicitly take into account the dependence of the sampling, inspection, and defective product costs on the chart parameters selected, some researchers have suggested the formulation of economic models as an alternative method to a purely statistical approach for designing the control charts.

The *economic design* approach to control charts advocates the determination of the control chart design parameters based on a cost-minimization model that takes into account all costs affected by the choice of these parameters (Lorenzen and Vance, 1986). The economic design of control charts for monitoring the process mean has been investigated extensively in the literature (see, e.g. Montgomery, 1980; Ho and Case, 1994a). In the literature on control charts employing an EWMA type statistic, several authors have explored the economic design of EWMA control charts to monitor the process mean (Ho and Case, 1994b; Montgomery et al., 1995). Park et al. (2004) extended the traditional economic design of an EWMA chart to the case where the sampling interval and sample size may vary depending on the current chart statistic. Tolley and English (2001) studied the economic design of a control scheme combining both EWMA and X-bar charts.

Another research stream in the literature has considered the joint economic design of mean and variation control charts. Although the joint economic design of *X-bar* and *R* (range) or *X-bar* and *S* (standard deviation) control charts has been studied by several researchers (e.g., Saniga, 1979; Rahim, 1989; Rahim and Costa, 2000; McWilliams et al., 2001), joint economic design of EWMA based control charts for monitoring both process mean and dispersion does not appear to have been previously investigated. We propose such a model that can be used to design a joint EWMA-based mean (μ) and variance (σ^2) control scheme. Our model is built upon the general cost function of Lorenzen