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Assessing process capability based on Bayesian approach with subsamples

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Abstract

Process capability indices (PCIs) have been widely used to measure the actual process information with respect to the manufacturing specifications, and become the common language for process quality between the customer and the supplier. Most of existing research works for capability testing are based on the traditional frequentist point of view and statistical properties of the estimated PCIs are derived based on the assumption of one single sample. In this paper, we consider the problem of estimating and testing process capability using Bayesian approach based on subsamples collected over time from an in-control process. The posterior probability and the credible interval for the most popular index C_{pk} under a non-informative prior are derived. The manufacturers can use the presented approach to perform capability testing and determine whether their processes are capable of reproducing product items satisfying customers' stringent quality requirements when a daily-based or weekly-based production control plan is implemented for monitoring process stability.

Keywords: Bayesian approach; Credible interval; Multiple samples; Process capability indices; Posterior probability

1. Introduction

Process capability indices (PCIs) have received considerable attention in quality assurance research and increased usage in process assessments and purchasing decisions (see Kotz and Johnson, 2002 for a good summary and discussion). Those indices provide common quantitative measures on manufacturing capability and production quality, which can be used by both customer and supplier as a reference when signing a contract. Therefore, PCIs are becoming powerful standard tools for quality report, particularly, at the management level around the world.

Proper understanding and accurate estimating of PCIs are essential for a modern company to maintain the status of a capable supplier. The approach by simply looking at the values of the estimated indices and then make a conclusion on whether the given process is capable, is highly unreliable since sampling errors have

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been ignored. As the use of PCIs grows more widespread, users are becoming educated and sensitive to the impact of the estimators and their sampling distributions, learning that capability measures must be reported in confidence intervals or via capability testing. Most of existing research works for capability testing have focused on the traditional frequency approaches. However, the sampling distributions are usually quite complicated, this makes establishing the exact confidence interval very difficult. Examples include Chan et al. (1988), Chou and Owen (1989), Chou et al. (1990), Bissell (1990), Zhang et al. (1990), Boyles (1991), Franklin and Wasserman (1992a,b), Kushler and Hurley (1992), Pearn et al. (1992), Subbaiah and Taam (1993), Nagata and Nagahata (1994), Chen and Hsu (1995), Tang et al. (1997), Hoffman (2001), Pearn and Shu (2003), Pearn and Lin (2002, 2004) and many others.

An alternative to the frequency approach is to consider the Bayesian statistical techniques. These techniques specify a prior distribution for the parameter of interest, in order to obtain the posterior distribution for the parameter. We then could make inferences about the parameter by using its posterior distribution given the sample data. It is not difficult to obtain the posterior distribution when a prior distribution is given, even when the form of the posterior distribution is complicated, as one can always use numerical methods or Monte Carlo methods to obtain an approximate but quite accurate interval estimate (Berger, 1985; Kalos and Whitlock, 1986). This is the advantage of the Bayesian approach over the traditional distribution frequency approach.

Cheng and Spiring (1989) proposed a Bayesian procedure for assessing process capability index C_p. Chan et al. (1988) applied a similar Bayesian approach to index C_{pm} under the assumption that the process mean μ is equal to the target value T. Shiau et al. (1999a) derived the posterior distributions for C_p^2, C_{pm}^2 under the restriction that process mean μ equals to the target value T, and for C_{pk}^2 under the restriction that the process mean μ equals to the midpoint of the two specification limits, M, with respect to the two priors (a non-informative and a Gamma prior). However, the restriction of $\mu = T$ or $\mu = M$ is not a practical assumption for many industrial applications. A nice Bayesian procedure for assessing process capability index C_{pm} relaxing the restriction on the process mean was proposed by Shiau et al. (1999b). They also applied a similar Bayesian approach for testing the index C_{pk} but under the restriction $\mu = M$. We note that in this case C_{pk} reduces to C_p. Subsequently, Pearn and Wu (2005a) considered the index C_{pk} for assessing process capability without restriction on the process mean.

In practice and in much of the quality control literature, the process performance is monitored and controlled by periodically collecting subsamples of data (i.e. based on the concepts of rational subgrouping). However, most of the results obtained so far regarding the statistical properties of estimated capability indices are based on the assumption of a single sample (see, e.g., Chou et al., 1990; Boyles, 1991, Kotz et al. (1993), Vännman and Kotz (1995), Spiring (1997), Vännman (1997), Pearn et al. (1998, 2004), Wright (2000), Zimmer et al. (2001), Pearn and Lin (2004)). To use estimators based on multiple samples and then interpret the results as if they were based on a single sample may result in incorrect conclusions, and vice versa (Vännman and Hubele (2003)). In order to use past in-control data from multiple samples to make correct decisions regarding process capability, the distribution of the estimated capability index based on multiple samples should be taken into account.

Therefore, in this paper we consider the problem of estimating and testing the most popular process capability index C_{pk} based on multiple samples collected over time for an in-control process. We propose accordingly a Bayesian procedure for capability assessing. The posterior probability p for which the process under investigation is capable, is derived. The credible interval, a Bayesian analogue of the classical lower confidence interval, can also be obtained. Practitioners can use the obtained results to determine whether their manufacturing processes are capable of reproducing products satisfying the preset process capability requirement when a daily-based or weekly-based production control plan is implemented for monitoring process stability.

2. Process capability analysis

2.1. Process capability index C_{pk} and process yield

PCIs provide common numerical measures for determine whether a process of reproducing items meeting the manufacturing specifications. The first process capability index appearing in the literature was the precision index C_p and defined as (see Juran, 1974; Sullivan, 1984, 1985; Kane, 1986)