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Discrete Optimization

A branch-and-bound algorithm for single-machine scheduling with batch delivery minimizing flow times and delivery costs

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Abstract

This paper addresses scheduling a set of jobs on a single machine for delivery in batches to customers or to other machines for further processing. The problem is a natural extension of minimizing the sum of flow times by considering the possibility of delivering jobs in batches and introducing batch delivery costs. The scheduling objective adopted is that of minimizing the sum of flow times and delivery costs. The extended problem arises in the context of coordination between machine scheduling and a distribution system in a supply chain network. Structural properties of the problem are investigated and used to devise a branch-and-bound solution scheme. Computational experiments show significant improvements over an existing dynamic programming algorithm.

Keywords: Scheduling; Single machine; Batch delivery; Branch-and-bound algorithm

1. Introduction

Scheduling groups of jobs on a single machine is a subject that has recently received growing interest, due to the desire for exploiting economies of scale. The relevant models are called family scheduling models, for which two alternative assumptions may apply. The first is batch availability, under which all the jobs forming a batch become available for later processing or dispatch only when the entire batch has been processed. Under the second possible assumption of job availability, a job becomes available once it has been processed. This paper adopts the first assumption.

An extensive survey of family scheduling models is available in Webster and Baker [11]. Potts and Kovalyov [10] present a review of scheduling with batching. Their work is particularly focused on considering the efficiency of dynamic programming algorithms for solving this type of problems.

One important class of batch availability models occurs when a machine requires setup time. Such

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models apply when jobs are partitioned into families according to similarity, so that there is no need for setup or changeover when a job follows another of the same family; i.e., a setup is required only between two different families and at the start of the schedule. In the case where there is just one family, the problem of minimizing the total flow time (F) is solvable by a backward dynamic programming algorithm that requires $O(n \log n)$ time, where n is the number of jobs [5]. However, most work on family scheduling under the batch availability assumption is concerned with minimizing the total weighted flow time (F_w). Albers and Brucker [1] proved that this problem is NP-hard, but is solvable in $O(n \log n)$ time in the special case where all jobs have the same processing time and are sequenced in the non-increasing order of their weights.

Mason and Anderson [9] propose a number of properties of the structure of the optimal solution of the problem of family scheduling under the batch availability with minimizing the total weighted flow time and derive a branch-and-bound algorithm for finding it. One of the most important aspects of their work is the proof that in the optimal sequence, batches are in order of non-decreasing weighted processing time (WPT). Computational results show that their algorithm can deal efficiently with problems of up to 30 jobs.

Also a branch-and-bound method that can solve problems of up to 50 jobs is provided by Crauwels et al. [6]. A special feature of their algorithm, which improves it in comparison with the algorithm of Masson and Anderson, is the use of a lower bound based on Lagrangian relaxation of the machine capacity constraint.

Cheng et al. [3] consider a problem in which n jobs of T different types are to be processed on a single machine. The items are to be batched, such that the jobs in each batch are of the same type. A setup time is incurred between batches. The batches are then to be sequenced to minimize the total weighted flow time. A dynamic programming algorithm that runs in $O(n^{m+1}/m^{m-1})$ is offered. Allahverdi et al. offer an extensive survey of scheduling problems involving setup [2].

Another important class of batch availability models occurs when the jobs are to be delivered to different customers or transferred to other machines in batches. In this case, no setup time is needed, but a delivery cost (delivery time) that depends on the customer is required for each batch. The problem is to batch and sequence batches such that the sum of flow times plus delivery costs is minimized. This class of problems is important within the framework of supply chain management. Yet, few works address it. Cheng et al. [4] consider a problem that arises when the objective is to minimize the sum of a function of the number of batches and job earliness penalties, where the earliness of a job is defined as the difference between the batch delivery date and the job completion time. A relation between this problem and parallel machine scheduling is established, which in turn makes it possible to reach complexity results and algorithms.

Hall and Potts [8] consider a variety of scheduling, batching and delivery problems that arise in an arborescent supply chain where a supplier makes deliveries to several manufacturers, who also make deliveries to customers. The objective is to minimize the overall scheduling and delivery cost, using several classical scheduling objectives. For each problem, they either derive an efficient dynamic programming algorithm, or demonstrate that it is intractable.

One of the problems identified by Hall and Potts is that of batching and sequencing on a single machine under the batch availability assumption, in order to minimize the sum of flow times plus delivery costs. Using an idea of Albers and Brucker [1] for a similar problem, Hall and Potts provide a forward dynamic programming algorithm that has $O(n^{m+1})$ complexity, where m is the numbers of job families, each of which consisting of all jobs destined for a particular customer. The Hall and Potts algorithm is thus exponential in the number of job families, which leaves the complexity status of the problem itself open.

In this paper, we consider this problem, study its structural properties, derive upper and lower bounds, offer a branch-and-bound scheme for solving it, and provide comparative computational results.

2. Problem definition

Let there be n jobs to be delivered in batches to m customers. The jobs are processed on a single machine that can process at most one job at a time and the processing time of job i is p_i. Each job is produced for one customer. A group of jobs forms a batch if they are all delivered to their customer at the same time. Let d_j denote the non-negative cost of delivering a batch to customer j. The objective is to minimize the total flow time or completion time