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Continuous Optimization

A note on the use of vector barrier parameters for interior-point methods

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Abstract

A key feature to ensure desirable convergence properties in an interior-point method is the appropriate choice of an updating rule for the barrier parameter. In this work we analyze and describe updating rules based on the use of a vector of barrier parameters. We show that these updating rules are well defined and satisfy sufficient conditions to ensure convergence to the correct limit points. We also present some numerical results that illustrate the improved performance of these strategies compared to the use of a scalar barrier parameter.

Keywords: Interior point methods; Primal-dual methods; Non-convex optimization; Central path

1. Introduction

Interior-point methods can be used to compute local solutions for nonlinear, and possibly non-convex, problems of the form

$$\begin{aligned} \min_{x} \quad & f(x) \\ \text{s.t.} \quad & c(x) = 0, \\ & x \geqslant 0, \end{aligned} \tag{1}$$

where $f: \mathbb{R}^n \mapsto \mathbb{R}$ and $c: \mathbb{R}^n \mapsto \mathbb{R}^m$. These methods have proved to be very successful for the solution of linear and general convex problems. Recently, a significant amount of effort has been devoted to extending these procedures to non-convex problems, see for example El-Bakry et al. [2], Gajulapalli [4], Gay et al. [5],

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Vanderbei and Shanno [15], Yamashita [17], Tits et al. [14], Moguerza and Prieto [10], among others. These methods proceed by (approximately) solving a sequence of equality-constrained problems of the form

$$\begin{aligned} \min_x \quad & f(x) - \bar{\mu} \sum_i \log x_i \\ \text{s.t.} \quad & c(x) = 0, \end{aligned} \tag{2}$$

for a sequence $\{\bar{\mu}^{(k)}\}$ of values of the scalar barrier parameter $\bar{\mu}$ such that $\bar{\mu}^{(k)} \to 0$.

An appropriate choice of values for the parameter $\bar{\mu}$ may have a significant impact on the practical performance of the algorithm, both on its convergence (a sequence that converges to zero at an excessively fast rate may imply numerical difficulties and lack of convergence), and its rate of convergence (a slowly convergent sequence will imply a slow algorithm). Several updating rules have been proposed in the literature, such as for example the widely used rule described in El-Bakry et al. [2].

In this paper we are interested in exploring the possibility of using a vector barrier parameter, $\mu \in \mathbb{R}^n$, to improve the practical convergence properties of the algorithm, and particularly its speed of convergence. A vector parameter should be better adapted to the possibly heterogeneous convergence behavior of the different variables in the problem. The use of vector parameters has been analyzed in the solution of linear problems by Nesterov and Nemirovskii [13] and Jansen et al. [8], among others. These approaches, although interesting from a theoretical point of view, did not result in major practical breakthroughs. A reason why these approaches have not been very successful in practice is that they increase the algorithmic complexity of the methods. Our goal is to design a robust and efficient updating strategy for a vector barrier parameter to be used within an interior-point method for the general nonlinear case.

As our interest focuses on this parameter, we will carry out the studies in this paper considering different definitions within the framework of the algorithm proposed by Moguerza and Prieto [10], an interior-point linesearch method using directions of negative curvature. The search directions are computed to approximate the solutions of the barrier problems, similar to (2),

$$\begin{aligned} \min_x \quad & f(x) - \sum_i \mu_i \log x_i \\ \text{s.t.} \quad & c(x) = 0. \end{aligned} \tag{3}$$

In the remainder of the paper we will use the bracketed superindex (k) to denote iterations and the subscript i to indicate each one of the components of the vector μ in (3).

The procedure to combine the directions is related to that in Moré and Sorensen [12]. A linesearch has been introduced in our algorithm as a mechanism to enforce good global convergence properties. We compute the iterates in such a manner that the value of an augmented Lagrangian merit function is decreased in each iteration. For problem (3) this merit function takes the form

$$L_A(x, \lambda; \rho, \mu) = f(x) - \sum_i \mu_i \log x_i - \lambda^T c(x) + \frac{1}{2} \sum_j \rho_j c_j(x)^2. \tag{4}$$

Under suitable assumptions the local minimizers for problem (3) are minimizers for this merit function, if all components of ρ are large enough and the vector λ corresponds to the optimal multipliers, see Bertsekas [1], for example.

The paper is organized as follows: In Section 2, we introduce the notation used in this work. Section 3 describes the general results that motivate our proposals. In Section 4, we present several updating strategies for the vector of barrier parameters μ and comment on some of their theoretical properties. Section 5 includes the results of a numerical comparison among the proposed strategies on a problem test set. Section 6 introduces some conclusions.

2. Notation and background

The first-order Karush–Kuhn–Tucker (KKT) conditions for problem (1) are: