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Discrete Optimization

Multiple voting location and single voting location on trees

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Abstract

We examine voting location problems in which the goal is to place, based on an election amongst the users, a given number of facilities in a graph. The user preference is modeled by shortest path distances in the graph. A Condorcet solution is a set of facilities to which there does not exist an alternative set preferred by a majority of the users. Recent works generalize the model to additive indifference and replaced user majority by γ-proportion.

We show that for multiple voting location, Condorcet and Simpson decision problems are Σ_2^p-complete, and investigate the approximability of the Simpson and the Simpson score optimization problem. Further we contribute a result towards lower bounds on the complexity of the single voting location problem.

On the positive side we develop algorithms for the optimization problems on tree networks which are substantially faster than the existing algorithms for general graphs. Finally we suggest a generalization of the indifference notion to threshold functions.

Keywords: Group decisions and negotiations; Facility location; Condorcet; Simpson score; Efficient graph algorithms

1. Introduction

Location problems on graphs are characterized as follows: An edge weighted graph models distances in a universe. Weighted nodes of the graph represent customers. The customers can be served by facilities which can be placed at the nodes of the graph. The goal is to find an optimal placement of the facilities. Several objective functions are in common use, e.g. maximum or average distance to the closest facility (*center* and *median* problem), or total sum of distances and costs for opening the selected set of facilities (*facility location* problem).

Voting problems are a means of modeling the process of finding compromises in a group of social individuals. Here, global decisions are often based on individual preferences which can be, more or less explicitely, treated as a formal election between alternative solutions. It is assumed that the resulting solutions are accepted by all participants and hence stable, since they are preferred by a significant majority of the users. Due to the nature of this model voting scenarios are often suitable to be solved by local algorithms, i.e., algorithms which are executed by an individual with a limited amount

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of information about the total scenario and a limited amount of communication with other individuals. Often local algorithms can be parallelized easily.

Voting location problems are a way to combine both lines of research: The static universe is modeled by a weighted graph, while the optimal placement of facilities is the result of an election process performed by the individual users. Here the user preferences are fully determined by the distances in the underlying graph. *Single* and *multiple* location problem distinguishes the cases where one or more facilities are to be placed, respectively.

The problems under investigation in this paper are as follows: The input graph specifies locations for users and facilities. There is a voting process among the users which aims to find an optimal placement of a set of facilities. The voting preference of individual users is based on the distance to the possible facility sets in the input graph. If a facility set is preferred over a concurrent facility set by more than half of the users, we say that the first set *dominates* the latter. A *Condorcet set* is a set of facilities which is not dominated by any alternative. The *Simpson score* of a facility set is the user preference of the strongest alternative; a low Simpson score can therefore be interpreted as a particular stable voting result. A *Simpson solution* is the most stable possible result, namely a facility set with minimum Simpson score. We refer the reader to the following chapter for a detailed definition of those notions.

Applications of such voting location problems can be found, e.g. in the area of facility location planning. In the classical facility location problem, decisions are performed based solely on a couple of abstract cost functions. These cost functions reflect mainly the view of the manufacturer, hence it is questionable whether the customer would be content enough to accept the implemented solutions. Voting location can be used to make statements about the *stability* of solutions: we not only seek for solutions minimizing the costs but also minimizing the attractiveness of alternative solutions. When such a solution is implemented then it can be assumed that it is widely accepted by the customers.

2. Problem definition and preliminaries

An instance of the *multiple voting location problem* (MVLP) is given by an undirected graph $G = (V, E)$ with finite node set V and positive edge weights $d\colon E \to \mathbb{R}^+$ inducing a distance function $d\colon V \times V \to \mathbb{R}_0^+ \cup \{\infty\}$ between pairs of nodes. Two subsets $U, L \subseteq V$ denote the *set of users* and the *set of possible locations*, respectively. We assume that $U = L = V$ unless otherwise stated. A nonnegative node weight function $w\colon U \to \mathbb{R}_0^+$ represents the number of users at nodes in the graph. Additionally the instance specifies a number $p \in \mathbb{N}$ $(1 \leqslant p \leqslant |L|)$ of facilities to place.

An instance of the *single voting location problem* (SVLP) is defined similarly with the restriction that $p = 1$.

2.1. Notation

We make use of the following standard notation: The *eccentricity* of a node $u \in V$ is defined as $e(u) := \max_{v\in V} d(u, v)$. The *radius* of a graph with node set V is defined by $\min_{u\in V} e(u)$. The *distance* between two node sets $V_1, V_2 \subseteq V$ is defined to be $d(V_1, V_2) := \min_{v_1\in V_1, v_2\in V_2} d(v_1, v_2)$.

If the input graph T is a tree, and $u, v \in V$ are two disjoint nodes, then by $T_u(v)$ we denote the subtree with root v hanging from v where the tree T is considered as rooted at u.

As said before we are only interested in solutions which consist of exactly p facilities. To this end, let $\mathscr{L}_p := \{X \subseteq L \mid |X| = p\}$ be the family of facility sets of cardinality p.

2.2. Condorcet and Simpson

We reflect the notation for Condorcet and Simpson solutions on MVLP instances from [1]: Let $u \in U$ be a user and $X, Y \in \mathscr{L}_p$ be two sets of facilities. The user u *prefers* Y over X if $d(u, Y) < d(u, X)$ and the user *is undecided* if these distances are equal. By

$$U(Y \prec X) := \{u \in U \mid d(u, Y) < d(u, X)\}, \tag{1}$$

we denote the set of users preferring Y over X. The weight of this set is denoted by

$$W(Y \prec X) := w(U(Y \prec X)). \tag{2}$$

Definition 2.1 (*Dominating solution*). Let $X, Y \in \mathscr{L}_p$. We say that solution Y *dominates* X, denoted by $Y \prec X$, if $W(Y \prec X) > \frac{1}{2} w(U)$.

In other words, Y is preferred over X by a majority of the users. Notice that the dominance relation is not necessarily a transitive relation.