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Decision Support

Optimal product bundling with dependent valuations: The price of independence

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ABSTRACT

In this paper we investigate the tactical problem of pricing a bundle of products when the underlying valuations of the bundle components are dependent. We use copula theory to model the joint density of reservation prices and provide analytical derivations for the prices under different bundling strategies and sharp bounds for the profit function. We discover that when only the bundle is offered and the marginal costs are relatively small, the seller is better off by bundling products that have a negative association between their valuations, while the converse is true when the marginal costs are relatively high. We also show that the net benefit of offering a full product line containing both the bundle and the components decreases for mild to strong associations between the component valuations, compared to offering just the bundle. Finally, we analyze how the typical literature assumption of independence of reservation prices impacts the seller's profitability when in fact the valuations are dependent, and find that this gap in profitability, which we call the "price of independence", can be arbitrarily large.

1. Introduction

Bundling, i.e. selling a package of individual products or services as one unit, is an extensively used marketing strategy in many industries. For example, online travel aggregators such as Expedia or Travelocity provide vacation packages consisting of hotel, airfare, car rental, and tickets to local attractions; content providers such as Condé Nast and HBO offer multi-platform access to their products, such as printed and digital, or TV and streaming; computer and mobile phone manufacturers pre-load hardware with various software packages; and telecommunication companies provide "triple-" or "quadruple-play" packages which include landline phone, broadband Internet, digital television, and wireless phone services. Bundling is widely used in practice because it acts essentially as a price-discrimination mechanism (Stigler, 1963), it reduces the buyers' heterogeneity (Schmalensee, 1984) and thus enables the seller to extract additional consumer surplus, as long as the seller has market power. Even when competition effects are present, a telecommunication company such as Verizon can leverage its quadruple-play package and make it very hard for a regional telecommunications company to compete in smaller markets. In this case, bundling is virtually a deterrent strategy for an established firm and acts as an entry barrier (Nalebuff, 2004).

In general, companies usually choose one of three possible bundling strategies (Adams & Yellen, 1976). Under *Pure Components* (PC) the seller chooses to sell only the components, but not the bundle, while under *Pure Bundling* (PB) she offers only the bundle for sale, but not the components. Finally, under *Mixed Bundling* (MB) she offers the bundle(s) as well as the components, separately, for sale. When in fact all possible combinations of bundles and separate components are offered, we say that the seller uses a *full mixed bundling* strategy and when only a subset of all the possibilities ends up being offered, then she uses a *partial mixed bundling* strategy.

One interesting—and generally hard—problem in bundling is how to price (optimally) the various bundles that the seller can offer. The difficulty of the problem stems from the different dimensions that can be considered in formulating an acceptable answer. For example, first consider the number of components, N, that can be bundled. Clearly, the full mixed bundling strategy is the most challenging to implement since the number of total packages (i.e. bundles plus components) that need to be priced equals $2^N - 1$. In contrast, the PC strategy requires pricing only the N components, while the PB strategy involves the pricing of a single package. In addition to the exponential number of decision variables, the mixed bundling strategy also involve an exponential number of pricing constraints, since a typical bundle price needs to be offered

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at a lower price than the sum of its component prices (otherwise the buyers can assemble the bundle for themselves). For these reasons, the majority of the bundling literature tends, mainly for tractability, to focus on $N = 2$ products. There are, however, notable exceptions. Hanson and Martin (1990) use a mathematical programming approach to compute optimal bundle prices as well as identifying which bundles should be offered, assuming that all reservation prices can be correctly identified. Bitran and Ferrer (2007) use a multinomial choice model to estimate what bundles should be offered, considering those offered by the competition. Ibragimov and Walden (2010) consider the problem of pricing a bundle consisting of a finite number of components with heavy tailed valuations, while Ferrer, Mora, and Olivares (2010) consider dynamic pricing of a line of interchangeable bundles consisting of a product and service.

The second difficulty that appears in bundle pricing is due to the possible dependence structure between consumers' valuations for the components. Additionally, consumers' valuation function for the bundle can be additive, sub-additive, or super-additive in the components, thus reflecting differently the substitutability/complentarity relationship of the components. For example, a conglomerate such as General Electric manufactures and sells under its Appliances division various household appliances such as washers and dryers, for which customers may have divergent preferences. A retailer such as H&M frequently bundles different clothing items together using intuitive heuristic pricing rules such as "buy one product and get the second for 50 percent off" (thus, enabling the consumer to combine items that could be either complements, such as a shirt and a pair of pants, or substitutes, such as two shirts of different colors.) Similarly, in a quadruple-play bundle, a fixed telephone landline and a mobile phone plan serve intrinsically the same basic communication need, but the flexibility afforded by each of these options is quite different. Hence, depending on their particular situations, consumers searching for a phone contract or to furnish their newly-purchased home could have either a positive or negative association in their valuation of the products.

Most of the extant bundle pricing literature (e.g. Venkatesh and Mahajan, 1993; Eckalbar, 2010; Bhargava, 2013, among others), primarily for analytical tractability reasons, has focused strictly on the analysis when goods have independent consumer valuations. Several notable exceptions are the papers of Venkatesh and Kamakura (2003) who look explicitly at pricing bundles of complements and substitutes (although their analytical results are limited to the pure bundling/pure components scenarios), Banciu, Gal-Or, and Mirchandani (2010) who examine the bundling of substitutes in a vertical market, Armstrong (2013), who examines the profitability of bundling with sub- and super-additive component valuations, and McCardle, Rajaram, and Tang (2007) who extend the analysis of independent valuations to the cases of either perfectly positive and negative correlated components (although their results are limited to uniform marginal distributions). Interesting papers that address bundling with dependent valuations are Schmalensee (1984) who examines the profitability of bundling under correlated valuations described by the Normal distribution and Chen and Riordan (2013), who explicitly model the dependence component structure using copula functions while examining the general question of bundle profitability.

The main research aim of this paper is to investigate a combination of the first two dimensions discussed above. Specifically, we focus on two things. First, we look at how firms should price the individual products/services as well as the bundle, when the demands for the components exhibit structural dependence under each of the aforementioned bundling strategies. This is important for a lot of industries where bundling is prevalent, such as traditional retail or e-commerce. In an online setting, finding good quality solutions is critical if the bundles are created on the fly—for example, travel aggregators such as Expedia or Orbitz who offer travel packages need to price these bundles in real time, while the results page is loading for the consumers visiting the site. Second, we examine what happens when, either for convenience or due to poor marketing research efforts, bundles are priced as if there were no dependence relationship among the components. Therefore, in order to make the analysis more tractable and to keep our results comparable with the existing literature, we will make the following assumptions throughout the paper: we limit the analysis to a single seller (monopoly) offering two products ($N = 2$); we assume that consumers have bundle valuations that are additive in the components.

A shortcoming of (incorrectly) assuming independent valuations is that it may lead to misspecified models (Jedidi, Jagpal, & Manchanda, 2003), and thus resulting in a forfeit of revenue. We show that although the association between the consumers' valuations of the bundle components may be rather weak (possibly even appearing as independent), correctly accounting for dependence (in either direction) can provide significant incremental revenue. Furthermore, modeling the relationship between the products via a generic dependence structure allows us to partially capture some of the substitutability/complementarity effects that have typically been modeled using a sub- or super-additive bundle valuation function (Eppen, Hanson, & Martin, 1991; Venkatesh & Kamakura, 2003). The main benefit of our approach is that, in general, it is easier to estimate the magnitude of these effects in practice, since sellers can usually measure the consumers' valuations for the components via marketing research tools, but measuring directly the bundle valuation *for all possible bundles* is not as easy a task.

In order to accommodate the dependence structure between the components, we draw on the framework of *copula* functions. A copula function represents a statistical construct that "couples" the two component distributions and synthetically creates the joint valuation distribution, from which one can derive the bundle valuation via convolution. This proposed approach is particularly attractive, since one can create many different copula functions, for any choice of component pairs. The flexibility of this approach becomes clear if we make the simple observation that constructing a joint density with different valuations is very difficult using traditional methods (e.g. the valuation for a product follows a Lognormal distribution, while the valuation for the second product may be Gamma distributed). Moreover, in a multivariate setting, there may be a lot of partial correlations that are impossible to capture analytically; the copula approach bypasses these difficulties and provides tractability.

While copula functions have been discovered since the 1940s, in the general business literature they have been relatively slow to percolate. A notable exception is finance where copula functions have been heavily used, in particular in the theory of quantitative risk management (Kakouris & Rustem, 2014), where modeling the default risk of portfolios of correlated assets is of utmost importance. Some other isolated exceptions include Clemen and Reilly (1999) (a decision analysis paper), and for our purposes the more relevant papers of Meade and Islam (2010), who model the time between inter-purchases, Danaher and Smith (2011), a comprehensive survey about the the applications of copula theory in marketing, and Chen and Riordan (2013), who, to the best of our knowledge, were the first to propose studying bundling with dependent valuations using copulas. In particular, our basic model setup (a monopolist selling two products and a bundle) and choice of copula functions are consistent with Chen and Riordan (2013). However, our research goals diverge significantly from theirs. The main thrust of Chen and Riordan (2013) is to extend the seminal work of McAfee, McMillan, and Whinston (1989) by establishing sufficient conditions which guarantee the (weakly) dominance of