European Journal of Operational Research 255 (2016) 545–558

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Returns to scale and most productive scale size in DEA with negative data

Biresh K Sahoo[a,1,*], Mohammad Khoveyni[b], Robabeh Eslami[c], Pradipta Chaudhury[d]

[a] Xavier Institute of Management, Xavier University, Xavier Square, Bhubaneswar, Odisha 751 013, India
[b] Department of Applied Mathematics, College of Basic Sciences, Yadegar-e-Imam Khomeini (RAH) Shahr-e-Rey Branch, Islamic Azad University, Tehran, Iran
[c] Department of Mathematics, Faculty of Technology and Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran
[d] Centre for Economic Studies and Planning, School of Social Sciences, Jawaharlal Nehru University, New Delhi 110 067, India

ARTICLE INFO

Article history:
Received 28 July 2015
Accepted 31 May 2016
Available online 8 June 2016

Keywords:
Data envelopment analysis
Returns to scale vs. Returns to growth
Level (static) efficiency vs. Growth (dynamic) efficiency
Most productive scale size
Negative data

ABSTRACT

Non-parametric evaluation of returns to scale of production units in standard DEA models becomes problematic when their underlying technologies involve negative data. The methodology recently offered by Allahyar and Rostamy-Malkhalifeh (2015) (hence after called ARM model) is of some help to deal with this issue. However, there are two shortcomings underlying the ARM model. First, it may not be capable of locating all the production units exhibiting constant returns to scale; and second, it is also not able to determine most productive scale size. In order to deal with these two shortcomings, the current paper contributes to the DEA literature in two ways. First, it makes a unifying attempt to propose a general non-radial DEA model to determine both the most productive scale size and the returns to scale characterizations of production units in the presence of negative data. Second, the proposed model can be adapted in a dynamic DEA technology setting to determine growth efficiency and returns to growth behavior of production units facing hyper competition in a new economy.

1. Introduction

One of the most important aspects in applied production analysis of organizational decision making units (DMUs) is *returns to scale* (RTS), which help determine pricing policies and market structure, and consequently, government policies toward both. It is therefore imperative that this concept be measured accurately. To assess efficiencies of a set of homogenous DMUs, it is necessary to identify the nature of RTS that characterize efficient production. In production economics, RTS in a multi-output and multi-input technology are defined as the maximal proportional increase in all outputs (α) resulting from a given proportional increase in all inputs (ζ). Constant returns to scale (CRS) prevail if $\alpha = \zeta$, increasing returns to scale (IRS) prevail if $\alpha > \zeta$, and decreasing returns to scale (DRS) prevail if $\alpha < \zeta$. For alternative definitions of RTS, see Baumol, Panzar, and Willig (1982) and Färe, Grosskopf, and Knox Lovell (1988).

Ever since the nonparametric methodology of data envelopment analysis (DEA) was introduced by Charnes, Cooper, and Rhodes (1978), the economic concept of RTS has been widely studied within the two broader frameworks of DEA.[2] The first framework by Färe, Grosskopf, and Knox Lovell (1985) proposes to determine RTS characterization of a DMU by considering the ratios of two radial efficiency measures under different RTS assumptions, i.e., the ratio of efficiency measure under CRS to either that under variable returns to scale (VRS) or non-increasing returns to scale (NIRS). The second framework that stems from the work by Banker (1984), Banker, Charnes, and Cooper (1984), and Banker and Thrall (1992) proceeds by examining tangential planes to the VRS-based DEA production frontier at a given point. This is done either by looking at the constant term that represents the intercept of that plane with the plane in which all inputs are set to zero or, by observing the weights of the corner points of the facet of the frontier associated with that plane.

This second framework is also extended to both additive and multiplicative DEA models. Unlike the radial CCR and BCC mod-

* Corresponding author. Tel.: +91 674 6647 735; fax: +91 674 2300995.
E-mail addresses: biresh@ximb.ac.in, biresh_00@yahoo.com, biresh.sahoo@hotmail.com (B.K. Sahoo), mohammadkhoveyni@gmail.com (M. Khoveyni), Roba_eslami@azad.ac.ir (R. Eslami), pradipta@mail.jnu.ac.in (P. Chaudhury).

[1] Biresh Sahoo was a Visiting Professor at CESP, Jawaharlal Nehru University, New Delhi whilst the first draft of this paper was developed.

[2] See, e.g., Färe and Grosskopf (1994), Banker, Bardhan and Cooper (1996), Banker, Chang, and Cooper (1996), Sueyoshi (1997, 1999), Seiford and Zhu (1998, 1999), Kerstens and Vanden Eeckaut (1999), Sahoo, Mohapatra and Trivedi (1999) Sahoo, Sengupta and Mandal (2007), Read and Thanassoulis (2000), Førsund and Hjalmarsson (2004), Tone and Sahoo (2003, 2004, 2005, 2006), among others.

http://dx.doi.org/10.1016/j.ejor.2016.05.065

els, the additive model by Cooper, Seiford, and Tone (2007) avoids the problem of choosing between input and output orientations. In case of multiplicative models (Banker & Maindiratta, 1986) where the piecewise linear frontiers usually employed in CCR and BCC models are replaced by the piecewise log-linear frontiers, RTS are obtained from the exponents of these piecewise log-linear functions for the different segments that form the underlying production frontier. Note that in both frameworks, RTS characterization of a DMU depends on whether input- or output-oriented model is used since different orientations identify different points on the frontier from which evaluations are effected.

Since the DEA technologies are not differentiable at extreme points, researchers suggest determining both right- and left-hand RTS at these extreme points (see, e.g., Banker & Thrall, 1992; Golany & Yu, 1997; Fukuyama, 2000, 2001, 2003; Førsund & Hjalmarsson, 2004; Hadjicostas & Soteriou, 2006; Førsund, Hjalmarsson, Krivonozhko, & Utkin, 2007; Podinovski, Førsund, & Krivonozhko, 2009; Podinovski & Førsund, 2010; Sahoo & Gstach, 2011; Sahoo & Tone, 2013, 2015; Zelenyuk, 2013; Eslami & Khoveyni, 2013; Sahoo & Sengupta, 2014; Sahoo, Zhu, & Tone, 2014; Sahoo, Zhu, Tone & Klemen, 2014; among others). However, in order to get away with two types (right-hand and left-hand) RTS, Boussemart, Briec, Peypoch, and Tavéra (2009) suggested, based on a homogeneous reference technology, an innovative $\alpha-$RTS method to determine RTS globally. Using a variant of the piecewise constant elasticity of substitution-transformation DEA technology by Färe, Grosskopf, and Njinkeu (1988), which is homogeneous of degree α, they determined $\alpha-$RTS globally for DMUs by exploiting the linkages between the input, output and graph distance functions.

In the aforementioned RTS studies, the DEA technologies are characterized by input-output data that are assumed to be non-negative. However, negative data may arise in several situations. There are some variables, which may naturally be interpreted as negative. Examples of such variables include pollutants, number of incorrect deliveries, number of complaints, number of undesirable orders delivered, sewage sludge, consignments for delivery, undesirable orders, percentage of consumers with negative brand perception before/after brand campaign, etc. Variables, when expressed in the form of some transformations, may actually turn negative. For example, a variable called profit, expressed as the difference between revenue and cost, may turn negative if cost exceeds revenue. Variables considered in growth forms such as increments of time/demand deposits, increments in sales/customers/deposits/income, growth of GDP per capita, etc. may be negative. Furthermore, while dealing with the estimation of piecewise log-linear technology, one may encounter negative data since the log transformation of values being less than 1 are always negative. One may refer to, e.g., Portela, Thanassoulis, and Simpson (2004), Pastor and Ruiz (2007), Sharp, Meng, and Liu (2007), Sahoo and Tone (2009), Sahoo, Luptacik, and Mahlberg (2011), Sahoo, Kerstens, and Tone (2012), Sahoo, Mehdiloozad, and Tone (2014), Mehdiloozad, Sahoo, and Roshdi (2014), among others, for several examples of applications with negative data.

It is therefore imperative that a special variant of RTS estimation model is necessary when the DEA technology involves negative data. As will be demonstrated in our method section (Section 2), it is because the VRS specification of a DEA technology involving negative data finds no DMUs exhibiting most productive scale size (MPSS), a term coined by Banker (1984), i.e., no DMU in this technology exhibits CRS. Second, this technology may also yield, for an inefficient DMU, a reference set, which consists of DMUs exhibiting IRS and DRS. This finding goes against the very property of a reference set by Tone (1996) that no reference set for any inefficient unit comprises of DMUs exhibiting IRS and DRS.

Though, there have been some attempts in the DEA literature (see, e.g., Portela et al., 2004; Sharp et al., 2007; Kerstens & Van de Woestyne, 2011, among others) on the direct treatment of negative data in efficiency evaluation, the determination of RTS against such technologies has rarely been exposed in DEA setting (a noticeable exception is Allahyar and Rostamy-Malkhalifeh (2015)). However, the approach by Allahyar and Rostamy-Malkhalifeh (2015) (henceforth the ARM approach) for determining both right- and left-hand RTS characterizations suffers from two shortcomings. First, their method may not be capable of properly determining the RTS characterizations; and second, their method is also not able to determine the MPSS. In order to deal with these two shortcomings, the current study proposes a non-radial DEA model to determine both MPSS, and right- and left-hand RTS characterizations.

As an innovative adaptation of our proposed methodology, we consider efficiency evaluation of the technology-intensive industries in a new economy in which a dynamic model of hyper-competition is considered more suitable. The new economy is differentiated from the old economy in terms of three features: (a) dynamic competition (growth or dynamic efficiency) as opposed to static competition (level or static efficiency); (b) innovation efficiency, access efficiency, and resource efficiency as opposed to technical and allocative efficiency; and (c) expanding markets. The growth and decay of these industries are primarily driven by efficiency in both static and dynamic (hyper) competition. While efficiency in static competition means technical and allocative efficiency, efficiency in hyper-competition requires innovation efficiency, access efficiency, and resource efficiency. On detailed account of these various efficiency concepts, see Sengupta (2003, 2004, 2007, 2011).

To assess growth efficiency (GE) of the high-tech companies, it is necessary to identify the nature of returns to growth (RTG) that characterizes the dynamic production frontier. The dynamic (intertemporal) technology set in a DEA setting is formed by considering all the feasible input and output growth vectors over a time period. The efficiency frontier resulting from this dynamic technology set is called the dynamic production frontier[3], a concept first introduced by Sengupta (2002) in a DEA setting to both theoretically illustrate and empirically analyze growth and decay behavior of the firms competing in hyper-competitive markets. Several applications based on this dynamic technology are found in studies by Sengupta, 2003, 2004, 2005a, 2005b, 2007, 2011; Sengupta and Sahoo, 2006 and Sengupta and Neogi, 2009. As regards the RTG, a concept coined by Sahoo, Kerstens, and Tone (2012), it relates growth rates in inputs to growth rates in outputs along a dynamic production frontier, and are defined as the maximal proportional increase in all output growth rates (say, η_1) resulting from a given proportional increase in all input growth rates (say, η_2). Constant RTG prevail if $\eta_1 = \eta_2$, increasing RTG prevail if $\eta_1 > \eta_2$, and decreasing RTG prevail if $\eta_1 < \eta_2$. The formulation of RTG characterization along a dynamic production frontier is an analogous of that of the RTS characterization defined along the boundary of a static production frontier relating inputs to outputs. Note that RTG cannot be synonymously used as *dynamic RTS*.[4]

[3] This literature must be distinguished from alternative models specifying the dynamics of production in a nonparametric context. For instance, the dynamic technology structure is analyzed and nonparametric tests are presented from a dynamic cost-minimizing perspective in Silva and Stefanou (2003). Silva and Stefanou (2007) develop nonparametric dynamic measures of efficiency in the short and the long run in this same framework. Nemoto and Goto (1999) and Ouellette and Yan (2008), among others, develop slight variations on this same theme.

[4] Dynamic RTS describe the same proportional input and output relations in a multistage framework where the black-box nature of technology is extended to two or more stages. In these so-called network models, the intermediate measures produced in the first stage are considered inputs to the second stage (Sueyoshi & Sekitani, 2005).