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Discrete Optimization

An ejection chain approach for the quadratic multiple knapsack problem

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ABSTRACT

In an algorithm for a problem whose candidate solutions are selections of objects, an ejection chain is a sequence of moves from one solution to another that begins by removing an object from the current solution. The quadratic multiple knapsack problem extends the familiar 0–1 knapsack problem both with several knapsacks and with values associated with pairs of objects. A hybrid algorithm for this problem extends a local search algorithm through an ejection chain mechanism to create more powerful moves. In addition, adaptive perturbations enhance the diversity of the search process. The resulting algorithm produces results that are competitive with the best heuristics currently published for this problem. In particular, it improves the best known results on 34 out of 60 test problem instances and matches the best known results on all but 6 of the remaining instances.

1. Introduction

The quadratic multiple knapsack problem (QMKP) (Hiley & Julstrom, 2006) extends the well-known 0–1 knapsack problem in two aspects. First, each knapsack possesses its own capacity, and each object can be assigned to at most one knapsack. Second, in addition to their individual values, objects have values in pairs that accrue to the total objective value when both objects in a pair are assigned to the same knapsack. The objective of QMKP is to fill the knapsacks with objects of maximum total value without exceeding the capacity of any knapsack. As a generalization and a combination of the multiple knapsack problem (Hung & Fisk, 1978) and the quadratic knapsack problem (Gallo, Hammer, & Simeone, 1980), QMKP is known to be NP-hard (Hiley & Julstrom, 2006).

Meta-heuristic algorithms (García-Martínez, Glover, Rodriguez, Lozano, & R., 2013; García-Martínez, Rodriguez, & Lozano, 2014; Hiley & Julstrom, 2006; Sundar & Singh, 2010) are powerful tools for handling the QMKP problem. Among these algorithms, local search is one of the most well-known techniques. However, local searches based on simple neighborhood moves may easily fall into the local optima. To overcome this drawback, a variable depth method called ejection chain approach examines a large search space by generating a sequence of interrelated simple moves to create compound moves. In the past two decades, ejection chain methods have been widely used to tackle a variety of challenging optimization problems (see Section 2.2). The current work is motivated by these applications to employ ejection chain methods for the QMKP.

Different from most local search heuristics that directly move from one solution to another, the ejection chain approach first moves to intermediate structures, called reference structures, before moving to another solution. During these procedures, a certain amount of infeasibility is imposed on the initial solution, which has to be ejected to obtain a new feasible solution. The ejection of infeasibility can be delayed to create a chain by moving to other reference structures. At each step of the chain, feasible solutions can be obtained by ejecting the infeasibility. Hence, the approach is termed ejection chain algorithm (ECA). The ejection chain approach can explore much larger search spaces in a compact manner than traditional local search heuristics based on simple neighborhood moves.

The main contributions of this paper are summarized as follows:

- To our knowledge, this work is the first to employ the ejection chain method to solve the QMKP. In addition, this technique has never been used to address other knapsack problems.
- Both greedy and random operators are embedded in the proposed ejection chain local search. This study also proposes an

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effective perturbation phase based on two specialized perturbation operators and an adaptive management mechanism.

- The performance of the ECA is tested on 60 benchmark instances that were extensively used in previous studies. The outcomes show the efficacy of this algorithm in terms of both solution quality and robustness. In particular, the ECA generates results competitive with those of state-of-the-art approaches presented in literature by improving the best known results on 34 instances and matching the best known results on all but 6 of the remaining instances.
- The effects of some important parameter settings and components of the proposed algorithm are analyzed.

The remainder of the paper is organized as follows: Section 2 presents the mathematical formulation of the QMKP and the related works. Section 3 describes the main components of the ECA. Section 4 presents the comprehensive computational results and comparisons between the ECA and some other best-performing algorithms in literature. The effects of several important components and the parameter settings of the proposed algorithm are analyzed in Section 5. Finally, Section 6 concludes this study and gives suggestions for future research directions.

2. Problem formulation and related works

2.1. Mathematical formulation of QMKP

The QMKP involves assigning a set of objects into knapsacks, such that the total profit of all objects in the knapsacks is maximized without violating the capacity constraint of any knapsack. It includes a set $N = \{1, \ldots, n\}$ of n objects and a set $M = \{1, \ldots, m\}$ of m knapsacks. Each object $i \in N$ has a profit value v_i and a weight w_i. Each pair of objects $i \in N$ and $j \in N$ $(i \neq j)$ has a profit value v_{ij}, while each knapsack $k \in M$ possesses a capacity C_k. Each object should be assigned to at most one knapsack k such that the total weight of the objects in each knapsack k does not exceed its capacity C_k. The value of an assignment of objects N to knapsacks M is the sum of the linear values of the included objects and the quadratic values of the object pairs that fall into the same knapsack. In the QMKP, the objective is to maximize the total profit value V_{sum}. The decision variable x_{ik} is 1 if object i is assigned to knapsack k; otherwise, the value is 0. Thus, QMKP can be formulated as follows:

$$Max \quad V_{sum} = \sum_{i=1}^{n}\sum_{k=1}^{m} x_{ik}v_i + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sum_{k=1}^{m} x_{ik}x_{jk}v_{ij}, \tag{1}$$

subject to

$$\sum_{k=1}^{m} x_{jk} \leq 1; \qquad j = 1, \ldots, n, \tag{2}$$

$$\sum_{j=1}^{n} x_{jk}w_j \leq C_k; \qquad k = 1, \ldots, m, \tag{3}$$

$$x_{jk} \in \{0, 1\}; \qquad j = 1, \ldots, n; \qquad k = 1, \ldots, m. \tag{4}$$

In the above formulation, objective function (1) aims to maximize the total profit value. Constraint (2) guarantees that each object can be assigned to at most one knapsack. Constraint (3) ensures that the total weight of any knapsack does not exceed its capacity. Constraint (4) imposes binary restrictions on decision variables.

2.2. Related works

In this section, related works on the algorithms for solving the QMKP and applications of the ejection chain approach are briefly reviewed.

Hiley and Julstrom (2006) presented the first study on QMKP in literature. The authors introduced three heuristic methods, namely, greedy heuristic, stochastic hill-climber method, and genetic algorithm. Greedy heuristic method filled one knapsack with one object at a time by choosing an unassigned object with the maximum profit/weight ratio. Hill-climber method removed some objects from the knapsacks, and then refilled the knapsacks by applying the afore mentioned greedy heuristic method. Genetic algorithm encoded candidate solutions as strings with lengths equal to the number of objects and employed the hill-climber method as its mutation operator. Singh and Baghel (2007) presented a new steady-state grouping genetic algorithm for QMKP. Saraç and Sipahioglu (2007) proposed another genetic algorithm to solve QMKP. They developed a specialized crossover operator to generate feasible solutions and presented two distinct mutation operators. Sundar and Singh (2010) introduced an artificial bee colony (ABC) algorithm based on the swapping of unassigned objects with already assigned ones. Experimental results demonstrated the superiority of the approach over several reference algorithms in terms of solution quality. The computational results obtained by Wang, Kochenberger, and Glover (2012) indicated that the branch and cut method can effectively solve the quadratic knapsack problem with multiple knapsack constraints.

Recently, García-Martínez et al. (2014) combined a novel local search procedure with an iterated greedy approach based on a tabu mechanism for QMKP. They extended the local search method proposed by Sundar and Singh (2010) to exchange any two objects assigned to different knapsacks. The tabu-based destruction mechanism stores the components that were recently removed from the incumbent solution via short-term memory and prevents these components from being added into the partial solution again. García-Martínez et al. (2013) also addressed the QMKP by using the strategic oscillation (SO) method. They defined critical levels for QMKP and designed strategies to exploit the constraint structure by effectively exploring solutions in the feasible and infeasible regions close to the constraint boundaries.

For applications of the ECA, Glover (1996) originally designed an ejection chain strategy to generate neighborhoods of compound moves with attractive properties for the traveling salesman problem. Rego and Roucairol (1996) employed an ejection chain procedure to generate compound moves to solve the vehicle routing problem. Yagiura, Ibaraki, and Glover (2004) embedded the ejection chain approach into neighborhood construction combined with tabu search to address the generalized assignment problem. Other successful and recent applications of this methodology are detailed in Burke and Curtois (2010); Kingston (2012); Lozano, Duarte, Gortázar, and R. (2012); Rego, James, and Glover (2010); Sevaux, Rossi, Soto, Duarte, and R. (2013).

3. Ejection chain algorithm

The ECA is initiated by selecting elements to undergo a change of state (e.g., to remove one object from its knapsack) (Glover, 1996). Then, it explicitly identifies a so-called reference structure, which is similar to but slightly different from a solution, for example violating some constraints or missing some elements. On the basis of several predefined transition rules, moves are generated from one reference structure to another, and back from reference structures to solutions. The transition rules, together with the reference structures, define the ejection neighborhood moves.

In general, the framework of the ECA consists of three phases: initial solution construction, ejection chain local search, and adaptive perturbation. More precisely, a greedy constructive algorithm first produces a promising solution as the initial solution. Then, it iteratively alternates between an ejection chain local search phase (to perform intensive search) and a perturbation phase (to discover