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Decision Support

Value of information in portfolio selection, with a Taiwan stock market application illustration

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ABSTRACT

Despite many proposed alternatives, the predominant model in portfolio selection is still mean–variance. However, the main weakness of the mean–variance model is in the specification of the expected returns of the individual securities involved. If this process is not accurate, the allocations of capital to the different securities will in almost all certainty be incorrect. If, however, this process can be made accurate, then correct allocations can be made, and the additional expected return following from this is the value of information. This paper thus proposes a methodology to calculate the value of information. A related idea of a level of disappointment is also shown. How value of information calculations can be important in helping a mutual fund settle on how much to set aside for research is discussed in reference to a Taiwan Stock Exchange illustrative application in which the value of information appears to be substantial. Heavy use is made of parametric quadratic programming to keep computation times down for the methodology.

1. Introduction

The problem of portfolio selection – on how to invest a sum of money across a series of assets for optimal return – continues to be a challenge, as it always has been ever since there has been accumulated wealth in the world. Let there be a beginning of a holding period and an end of the holding period. Also, let r_i be the return on asset i over the holding period, and w_i be the proportion of initial capital invested in asset i at the beginning of the holding period, and held in asset i throughout the holding period. With the goal being to maximize end of period wealth, and with constraints in canonical form, the problem of portfolio selection is

$$\max\ r_p = \sum_{i=1}^{n} r_i w_i$$
$$s.t. \sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0 \quad \text{for all } i \tag{1}$$

where r_p is the return on one's capital over the holding period, n is the number of securities eligible for inclusion in a portfolio, and the sum-to-one constraint along with the nonnegativity restrictions define the feasible region in decision space. In the model, r_p is portfolio return, and with the w_i weights arranged in the form of $\mathbf{w} = (w_1, \ldots, w_n)$, $\mathbf{w}$ is called a fund allocation vector.

The problem looks innocuous enough, as it appears to be a linear programming problem, which in fact it is, except for the objective function. The difficulty in the objective function is that the r_i, the returns of the individual securities over the holding period, are random variables, and hence r_p is a random variable. Portfolio selection is thus the problem of maximizing the random variable of portfolio return – but to do so it is necessary to make decisions on the w_i at the beginning of the holding period based upon the values of the r_i that are not known until the end of the holding period. This makes Model (1) a stochastic programming problem. In this form, the problem of portfolio selection has been much discussed and analyzed. Thousands of papers have been written on the problem as the basic model can take on many related forms.

As defined by Caballero, Cerdá, Muñoz, Rey, and Stancu-Minasian (2001), if in a programming problem some of the parameters take unknown values at the time of making a decision, and these parameters are random variables, then the problem is a stochastic programming problem. Stochastic programming problems are notoriously difficult to solve, and solution methods are usually developed based upon the type of problem being considered (Beraldi, Violi, & Simone, 2011; Shapiro & Philpott, 2007).

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One often contemplated approach is to employ interpretations and assumptions so as to strive for an equivalent deterministic formulation that can be solved in a reasonably straightforward fashion.

Through reasoning such as overviewed in many places as in Huang and Litzenberger (1988), it is generally accepted that investors are expected utility maximizers. Under this assumption, and where U is the investor's utility function, Model (1) can be rewritten equivalently as

$$\max E[U(r_p)]$$
$$s.t. \sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0 \quad \text{for all } i \tag{2}$$

An advantage of Model (2) is that all random variables have been cleared from the formulation. With investors assumed to possess declining marginal utility, U is at least known to be concave and increasing.

Two schools of thought have evolved on how to address Model (2) with its expected utility objective function. One is to try to acquire enough information about the decision maker's preferences to enable the creation of an optimization problem that can be solved directly for an optimal portfolio. The "safety first" strategy of Roy (1952) is an example of this approach. More recent examples involving a range of techniques, although in the multi-criteria arena, can be found, for instance, in Ballestero and Romero (1996), Arenas Parra, Bilbao Terol, and Rodríguez Uría (2001), Bilbao-Terol, Pérez-Gladish, Arenas-Parra, and Rodríguez-Uría (2006), Abdelaziz, Aouni, and El-Fayedh (2007), Fang, Lai, and Wang (2008), and Aouni, Colapinto, and La Torre (2014). But these techniques are difficult because the setting up of the optimization problem generally requires more knowledge about the optimal solution to be found than is possible beforehand. The other school of thought involves parameterizing U and then attempting to solve Model (2) for all unknown values of U's parameter(s).

Now, if U is quadratic, which is a common assumption in portfolio selection, there is only one parameter, and it is not difficult to show, as in many places including Steuer, Qi, and Hirschberger (2007), that $E[U(r_p)]$ is a function of the mean and variance of r_p in the form of

$$E(r_p) - \frac{1}{t}V(r_p) \tag{3}$$

where t is a risk tolerance parameter. With (3) concave and increasing, all potentially optimizing solutions of Model (2), with (3) substituted for $E[U(r_p)]$, can be obtained by computing all efficient (E, V) mean–variance combinations that occur in the following two-objective program:

$$\max E = E(r_p)$$
$$\min V = V(r_p)$$
$$s.t. \sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0 \quad \text{for all } i \tag{4}$$

Recognizing that the two objectives are to be optimized simultaneously, an (E, V) combination is efficient if and only if it is not possible to improve one of the criteria without deteriorating the other. Putting (4) into practice, we have

$$\max E = \sum_{i=1}^{n} \mu_i w_i$$
$$\min V = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i \sigma_{ij} w_j$$
$$s.t. \sum_{i=1}^{n} w_i = 1$$
$$w_i \geq 0 \quad \text{for all } i \tag{5}$$

where μ_i is the expected return of the ith security (that is, of the r_i random variable), σ_{ii} is the variance of r_i, and the σ_{ij}, $i \neq j$, are the covariances of the random variables r_i and r_j over the holding period. In bi-criterion format, this is the famous mean–variance model of Markowitz (1952), and as prescribed by Markowitz, the approach is as follows:

1. Specify Model (5) with all of its required μ_i, σ_{ii} and σ_{ij} values.
2. Solve Model (5) for all efficient (E, V) combinations and the fund allocation solution vector **w**, as a function of V, pertaining to them. Methods for doing this go back to Markowitz (1956).
3. Display the efficient (E, V) combinations in the form of a graph, called the efficient frontier.
4. Have the investor select from the efficient frontier his or her most preferred (E, V) combination.
5. For this (E, V) combination, retrieve from the **w** of Step 2 the specific portfolio composition corresponding to the V of the selected (E, V) combination. Provided all has been carried out accurately, this then is the investor's optimal portfolio.

As seen, the efficient frontier is central to the approach. This is because the efficient frontier displays precisely all efficient (E, V) combinations. That is, if a particular fund allocation vector can potentially be an optimal solution of Model (2), its (E, V) combination will be on the efficient frontier, and conversely, if a particular fund allocation vector cannot be an optimal solution of Model (2), its (E, V) combination will not be on the efficient frontier.

The success of Markowitz's mean–variance approach is often attributed to its mathematical tractability, but there are other reasons. One is that the approach allows different investors to have different optimal portfolios. Another is that, because one's optimal portfolio is usually only recognized as such after seeing that everything else is worse, the approach's efficient frontier lets one see the "everything else." However, a caveat comes with the approach.

While the evolution of Model (5) represents considerable achievement with regard to theory, the model is in fact a monster with regard to its demands for data. That is, for an upcoming holding period, the model needs n expected returns, n variances, and $(n^2 - n)/2$ covariances. This is a lot of information, and there may be no good way to get all of it. Hence there is a legitimate worry that errors in the values used for at least some of these quantities will propagate through Model (5) and affect the resulting "optimal" solution.

Fortunately, the σ_{ii} and the σ_{ij} do not create any especial difficulties as they are readily estimated from historical data and tend to be stable from holding period to holding period. However, as brought into sharp relief by Best and Grauer (1991), the μ_i are a different story. Not only are the μ_i lacking in the persistence of the variances and covariances (see DeMiguel & Nogales, 2009; Kan & Smith, 2008; Siegal & Woodgate, 2007), but as shown in Chopra and Ziemba (1993), at a risk tolerance of 50, errors in the μ_i are about 11 more serious than errors of the same relative size in the