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Continuous Optimization

A fuzzy goal programming approach to multi-objective optimization problem with priorities

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Abstract

In goal programming problem, the general equilibrium and optimization are often two conflicting factors. This paper proposes a generalized varying-domain optimization method for fuzzy goal programming (FGP) incorporating multiple priorities. According to the three possible styles of the objective function, the varying-domain optimization method and its generalization are proposed. This method can generate the results consistent with the decision-maker (DM)'s expectation, that the goal with higher priority may have higher level of satisfaction. Using this new method, it is a simple process to balance between the equilibrium and optimization, and the result is the consequence of a synthetic decision between them. In contrast to the previous method, the proposed method can make that the higher priority achieving the higher satisfactory degree. To get the global solution of the nonlinear nonconvex programming problem resulting from the original problem and the varying-domain optimization method, the co-evolutionary genetic algorithms (GAs), called GENOCOPIII, is used instead of the SQP method. In this way the DM can get the optimum of the optimization problem. We demonstrate the power of this proposed method by illustrative examples.

Keywords: Goal programming; Fuzzy goal programming; Multi-objective optimization; Priorities; Co-evolutionary genetic algorithm

1. Introduction

Goal programming (GP), firstly introduced by Charnes and Cooper in the early 1960's [1], is a useful method for DM to consider simultaneously many goals for a satisfactory solution. The first application quickly demonstrated its interests in a number of areas, and later numerous variants and a number of

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impressive applications followed. As a robust tool for multi-objective decision-making (MODM) problem [24], GP has been studied extensively for the past 35 years.

In the multi-criteria setting the special characteristic of GP models is the way the decision criteria is dealt with. Instead of the direct evaluation of the criteria outcomes, GP models explicitly introduce the desired target value for each criterion, and optimize the deviations of the criteria outcomes from these goals. The solution depends on the metrics used for the deviations and as well as the weighting method of the different goals. There are two common weighting methods. The first one is the fixed ordering of goals. In practice, this is implemented by searching a lexicographic minimum of the ordered deviation vector [2]. The second one is the use of weights on goals and the minimization of the weighted sum of goal deviations [3]. Sometimes, the minimization of the maximum deviation is also used as in [4]. The GP approach of multi-criteria problems has received increasing interest due to its modeling flexibility and conceptual simplicity [5].

However, determining precisely the goal value of each objective is difficult for DM, since possibly only partial information is known. To incorporate uncertainty and imprecision into the formulation, the fuzzy set theory, initially proposed by Zadeh in 1965 [6] is introduced in the field of conventional MODM problems, where aspiration level of objectives are assigned in an imprecise manner. According to the fuzzy theory, the inaccurate objectives and constraints are represented by certain kind of membership functions [7], for instance, the triangle-like or trapezoid-like membership functions, we call the inaccurate objectives and constraints as fuzzy objectives and constraints. The concept of fuzzy programming (FP) in a general level was first proposed by Tanaka et al. [8] under the framework of fuzzy decision of Bellman and Zadeh [7]. After that, Zimmermann [9] utilized FP approach to linear programming (LP) with several objectives. Introducing fuzzy uncertainty and imprecision into the goal programming problems, Narasimhan [10] initially presented fuzzy goal programming (FGP) by using membership functions, and some relative work was done [11–13].

GP and FP are two approaches to solve the vector optimization problem by reducing it to scalar formulation. Both of them need an aspiration level for each goal. These aspiration levels are determined either by the DM or the decision analyst. In addition to the aspiration levels of the goals, FP needs admissible violation constants for each goal. The larger violation (or tolerance) of the goal indicates the less important of this goal. It can be proved that every fuzzy linear program has an equivalent weighted linear goal program where the weights are the reciprocals of the admissible violation constants [14]. In general, every FP is a GP with some weights assigned to the deviational variables in the objective function, where the FP has fuzziness in the aspiration level, i.e. to get a solution that makes the objectives as close as possible to a specific goal within a certain limit. In this paper, we use the FP method to solve the GP problem with different priorities, and the results of the examples shows the power of this method.

In GP problem, it is practical to consider that there are different importance and priorities of the goals, and a lot of research is done with this problem. The priority structure is assigned by using different methods, such as fuzzy analytical hierarchy process [15]. Hannan [16,17] and Tiwari et al. [18] have used evaluation function method to aggregate different goal functions, with various weights for different goals in order to reflect the relative importance of the goals. When augmenting some weights to further emphasize the goal, however, opposite results may sometimes come out since the goal value is not a monotone function of the weights and the constraints exist. Furthermore, conventional fuzzy goal programming method [18–20] is used when the DM has a priority order toward different goals. Whereas, this method has low computation efficiency since many sub-problems are solved in sequence. The interactive method has been introduced into this problem and studied by many scholars [22,30], which requires DM must stand by and make important preferences at each step of the optimization process. The GAs are also introduced in this problem [31,32] in spite of heavy burden of computation.

Chen et al. [25] proposed an approach using an additive model to solve this problem, and when the goals are assigned to different levels, the comparison of different memberships of the goals are added as crisp constraints. The reason that we do not apply this method is that: (1) the added constraints are too strict for