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Continuous Optimization

A dual-based algorithm for solving lexicographic multiple objective programs

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Abstract

In this paper, we propose an algorithm for solving lexicographic multiple objective programs based upon duality theorem. In the existing algorithm, we should solve several linear programming problems (LPPs); therefore if, in particular, there are several objective functions, this method is not worthwhile from the viewpoint of computation. But in our new algorithm we just solve one LPP.

Keywords: Duality; Lexicographic; MOLP; Linear programming

1. Introduction

Sometimes in optimization problems we encounter problems with several objective functions. One group of these problems, namely, multiple objective linear programming (MOLP) problems are in the following form:

$$\begin{aligned} \max \quad & \mathbf{Cx}; && \mathbf{C} \in \mathbb{R}^{r\times n}, \\ \text{s.t.} \quad & \begin{cases} \mathbf{Ax} \leqslant \mathbf{b}; \\ \mathbf{x} \geqslant \mathbf{0}; \end{cases} && \begin{matrix} \mathbf{A} \in \mathbb{R}^{m\times n}, \\ \mathbf{x} \in \mathbb{R}^{n}. \end{matrix} \end{aligned}$$

A particular form of MOLP, namely, lexicographic multiple objective linear programs (LMOLP), in which the objective functions are ordered by their degree of priority, is as follows:

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$$\begin{aligned} &\max \quad \{\mathbf{c}_1\mathbf{x}, \mathbf{c}_2\mathbf{x}, \ldots, \mathbf{c}_r\mathbf{x}\} \\ &\text{s.t.} \quad \begin{cases} \mathbf{Ax} \leqslant \mathbf{b} \\ \mathbf{x} \geqslant \mathbf{0} \end{cases} \end{aligned}$$

The optimal solution of this problem is called preemptive optimal solution. The existing method for solving such problems is the lexicographic method. This method maximizes $\mathbf{c}_1\mathbf{x}$ over the feasible space, and among all alternative optimal solutions maximizes $\mathbf{c}_2\mathbf{x}$, and among all continuing alternative optimal solution maximizes $\mathbf{c}_3\mathbf{x}$, and so on, until the final problem of maximizing $\mathbf{c}_r\mathbf{x}$ over continuing alternative optimal solutions. It can be observed that this method leads to solving several LPPs; therefore if, in particular, there are several objective functions, this method is not worthwhile from the viewpoint of computation.

In this paper, we propose an algorithm that can find the optimal solution by solving a single LPP.

This paper is organized as follows: Section 2 contains some definitions and theorems used in this paper about LMOLP problems. In Section 3, we introduce the new algorithm. In Section 4, the modified phase-I problem is introduced to reach an initial solution for initiating the new algorithm. In Section 5, we present a numerical example, and finally in Section 6 conclusions are discussed.

2. Some definitions and theorems about LMOLP

Before introducing our new algorithm, providing some definitions and theorems seems in order.

Consider the following problems:

Problem (I)

$$\begin{aligned} &\max\{\mathbf{c}_1\mathbf{x}, \mathbf{c}_2\mathbf{x}, \ldots, \mathbf{c}_r\mathbf{x}\} \\ &\text{s.t.} \quad \begin{cases} \mathbf{Ax} \leqslant \mathbf{b} \\ \mathbf{x} \geqslant \mathbf{0} \end{cases} \end{aligned}$$

Problem (II)

$$\begin{aligned} &\max \; M^{r-1}\mathbf{c}_1\mathbf{x} + M^{r-2}\mathbf{c}_2\mathbf{x} + \cdots + M\mathbf{c}_{r-1}\mathbf{x} + \mathbf{c}_r\mathbf{x} \\ &\text{s.t.} \quad \begin{cases} \mathbf{Ax} \leqslant \mathbf{b} \\ \mathbf{x} \geqslant \mathbf{0} \end{cases} \end{aligned}$$

in which $\mathbf{A} \in \mathbb{R}^{m\times n}$ and $\mathbf{c}_t \in \mathbb{R}^{1\times n}$ $(t = 1, 2, \ldots, r)$.

Definition 1. Problem (I) is unbounded if either $\mathbf{c}_1\mathbf{x}$ has an unbounded optimal value on the feasible region or there exists a $t \in \{2, \ldots, r\}$ such that $\mathbf{c}_t\mathbf{x}$ has an unbounded optimal value over the alternative optimal solutions of previous objective functions.

Theorem 2. *Problem* (I) *is unbounded if and only if there exists an* $\overline{M} > 0$ *such that for each* $M \geqslant \overline{M}$ *problem* (II) *has an unbounded optimal value.*

Proof. Assume that problem (I) is unbounded. In this case, if $\mathbf{c}_1\mathbf{x}$ has an unbounded optimal value over the feasible region, then there exists a $\mathbf{d} \neq \mathbf{0}$ such that $\mathbf{c}_1\mathbf{d} > 0$, $\mathbf{Ad} \leqslant \mathbf{0}$ and $\mathbf{d} \geqslant \mathbf{0}$. It can be obviously shown that there exists a big enough $\overline{M} > 0$ such that for each $M \geqslant \overline{M}$, $M^{r-1}\mathbf{c}_1\mathbf{d} + M^{r-2}\mathbf{c}_2\mathbf{d} + \cdots + M\mathbf{c}_{r-1}\mathbf{d} + \mathbf{c}_r\mathbf{d} > 0$.

Since $\mathbf{Ad} \leqslant \mathbf{0}$, $\mathbf{d} \geqslant \mathbf{0}$ and $\mathbf{d} \neq \mathbf{0}$, for each $M \geqslant \overline{M}$ problem (II) has an unbounded optimal value.

On the other hand, if $\mathbf{c}_1\mathbf{x}$ has a bounded optimal value, assume that t is the smallest index such that $\mathbf{c}_t\mathbf{x}$ has an unbounded optimal value over the alternative optimal solutions of previous objective functions. This means that the following problem has an unbounded optimal value