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Discrete Optimization

A branch-and-price algorithm for scheduling parallel machines with sequence dependent setup times

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Abstract

We consider the problem of scheduling n independent jobs on m unrelated parallel machines with sequence-dependent setup times and availability dates for the machines and release dates for the jobs to minimize a regular additive cost function. In this work, we develop a new branch-and-price optimization algorithm for the solution of this general class of parallel machines scheduling problems. A new column generation accelerating method, termed "*primal box*", and a specific branching variable selection rule that significantly reduces the number of explored nodes are proposed. The computational results show that the approach solves problems of large size to optimality within reasonable computational time.

Keywords: Parallel machines scheduling; Setup times; Column generation; Branch-and-price

1. Introduction

The study of parallel machine problems is relevant from both the theoretical and the practical points of view. From the practical point of view, it is important because we can find many examples of the use of parallel machines in the real world. The motivation for this work was a real textile industry problem involving the production of tissues of different colors on unrelated parallel machines (the production bottleneck) with long sequence dependent setup times and availability dates for the machines and release dates for the

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jobs. From the theoretical point of view, it is a generalization of the single machine problem and a particular case of problems arising in flexible manufacturing systems. For a literature review on parallel machines, see Hoogeveen et al. (1997).

Column generation is used to solve models obtained by a Dantzig–Wolfe decomposition (Dantzig and Wolfe, 1960). For a survey on its use to solve integer programming problems, see Barnhart et al. (1998). A recent publication on column generation is Lübbecke and Desrosiers (in press). It has been also successfully applied to solve parallel machines scheduling problems, as follows.

van den Akker et al. (1999) studied the problem of minimizing the total weighted completion time of n jobs on m identical parallel machines. The problem was formulated as a set-covering problem with an exponential number of binary variables, n covering constraints, and a single side constraint. The computational results show that the lower bound is singularly strong, and that the outcome of the linear program is often integer. Their algorithm solves problems with $n = 100$ and $m = 10$ within reasonable computational time.

Chen and Powell (1999a) considered the problem of scheduling n jobs on m identical, uniform, or unrelated machines with two particular objectives: to minimize the total weighted completion time and to minimize the weighted number of tardy jobs. They first formulate these problems as integer programs, and then reformulate them, using a Dantzig–Wolfe decomposition, as set partitioning problems. Each column represents a machine schedule, and is generated by solving a single machine subproblem. Branching is conducted on the variables of the original integer formulation. The computational results indicate that this approach is promising and capable of solving problems with $n = 100$ and $m = 20$ for the first objective, and $n = 100$ and $m = 10$ for the second objective.

Chen and Powell (1999b) proposed an approach for the problem of scheduling n jobs with an unrestrictive large common due date on m identical parallel machines to minimize the total weighted earliness and tardiness. The problem was first formulated as an integer program and then reformulated as a set partitioning problem with side constraints. Columns represent partial schedules on single machines and are generated by solving two single machine subproblems. A branch-and-bound algorithm is used to find an optimal integer solution for the problem. The computational results show that the algorithm solves problems with up to 60 jobs in reasonable time.

Chen and Lee (2002) considered the problem of scheduling a set of independent jobs on identical parallel machines to minimize the total earliness-tardiness penalty of the jobs. All the jobs have a given common due window. The problem is formulated as a set partitioning type problem. The computational results show that the approach solves problems with $n = 40$ and $m = 6$ within a reasonable computational time.

Chen and Powell (2003) studied the multiple job families scheduling problem on identical parallel machines, with sequence dependent or sequence independent setup times, to minimize the total weighted completion time of the jobs. The computational results show that the approach solves problems with 8 families, $n = 40$ and $m = 6$ within reasonable computational time.

In this work, we develop a new branch-and-price optimization algorithm for the problem of scheduling a set of independent jobs, with release dates and due dates, on unrelated parallel machines with availability dates and sequence-dependent setup times, to minimize the total weighted tardiness. This problem is strongly NP-hard, because its special case $P\|\sum w_jT_j$ is known to be strongly NP-hard, even when $m = 1$ (Lenstra et al., 1977).

The problem considered here is more general, and has a more complex structure than the parallel machines scheduling problems found in the literature: its binary integer model has a larger scale, since machines are non-identical; the subproblems are more difficult to handle, because there are availability dates for the machines, release dates for the jobs and sequence-dependent setup times. Furthermore, it lacks the structural properties that were essential to the success of the works mentioned above. In fact, the identification of structural dominance properties (usually corresponding to some job ordering restrictions) is a