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Analyzing the effect of the inventory policy on order and inventory variability with linear control theory

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Abstract

In this paper we apply linear control theory to study the effect of various inventory policies on order and inventory variability, which are key drivers of supply chain performance. In particular, we study a two-echelon supply chain with a stationary demand pattern under the influence of three inventory policies: an inventory-on-hand policy that bases orders on the visible inventory at an installation, an installation-stock policy that bases orders on the inventory position (on-hand plus on-order inventory) at an installation, and an echelon-stock policy that bases orders on the inventory position at that installation and all downstream installations. We prove analytically that the inventory-on-hand policy is unstable in practical settings, confirming analytically what has been observed in experimental settings and in practice. We also prove that the installation-stock and echelon-stock policies are stable and analyze their effect on order and inventory fluctuation. Specifically, we show the general superiority of the echelon-stock in our setting and demonstrate analytically the effect of forecasting parameters on order and inventory fluctuations, confirming the results in other research.

Keywords: Supply chain management; Control theory; Multi-echelon inventory control; Echelon-stock policy; Bullwhip effect

1. Introduction

In a supply chain, inventory levels are controlled by inventory policies. Simple inventory policies base order decisions on local information, whereas more advanced inventory policies utilize information across the supply chain. The type of inventory policy has a significant effect on the variability of order quantities

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and inventory levels at the various stages of a supply chain. In particular, high order variability causes companies to hold excess capacity, to use overtime production, and to use premium shipping. High inventory variability motivates high levels of safety stock and makes large inventory costs unavoidable. Altogether, high variability is a main driver of supply chains performance [1] and so it is important to analyze the effect of inventory policies on order and inventory. We do this by applying linear control theory.

We consider three inventory policies, a simple inventory-on-hand policy and two base-stock policies. The inventory-on-hand policy bases order decisions on the amount of inventory that is available on hand. This type of inventory policy is clearly sub-optimal, but might be expected in practice. The first base-stock policy we analyze is an installation-stock policy. It bases order decisions on installation stock, which is the amount of inventory on hand at an echelon plus the amount of inventory that is on order from the preceding echelon. The second base-stock policy we analyze is an echelon-stock policy that bases order decisions on echelon stock, which is the sum of installation stock at an echelon and the installation stock of all downstream echelons. An additional difference between the installation-stock policy and the echelon-stock policy is that end-customer demand is known only to the downstream echelon under an installation-stock policy, whereas it is shared with all echelons under an echelon-stock policy.

The objective of our analysis is to identify the effect of the inventory policy on order and inventory variability. We prove that a supply chain that operates under an inventory-on-hand policy is not stable for positive lead times, which implies that even small fluctuations in demand result in uncontrollable fluctuations of order quantities and inventory levels. Therefore, this policy is not suitable for controlling inventory and we do not pursue it further. We also analyze the stability of supply chains that operate under installation-stock or echelon-stock policies. We prove that these supply chains are stable for arbitrary lead times which implies that both policies are suitable for controlling inventory. We show that although neither policy dominates the other for all demand patterns, the echelon-stock policy outperforms the installation-stock policy with respect to some important performance measures, white-noise amplification and worst-case amplification. White-noise amplification measures the effect of normally distributed demand on order and inventory fluctuations. Worst-case amplification measures the maximum order and inventory fluctuations in response to any stationary demand.

The contributions of this paper are threefold. First, we prove that supply chains that operate under inventory-on-hand policies are not stable in contrast to supply chains that operate under installation-stock and echelon-stock policies, which are stable. Second, we model the echelon-stock policy using linear control theory and derive closed-form expressions for several performance measures. Third, we prove the superiority of the echelon-stock policy over the installation-stock policy for the important performance measures white-noise and worst-case amplification. We also show several properties of the inventory policies, for example, that order quantities and inventory level oscillations are increasing in lead time and that improving the responsiveness of the forecasts increases variability.

Our research builds on two lines of literature, the literature on stochastic inventory models and the literature on linear control theory. The inventory models we use in this paper are based on the literature on stochastic inventory models. The base-stock policy for a single-echelon system was developed by Veinott [34]. The echelon-stock policy for a multi-echelon supply chain was proposed by Clark and Scarf [8]. Axsäter and Rosling [2] discuss installation-stock and echelon-stock policies and investigate circumstances under which the echelon-stock policy is superior. For an overview on stochastic inventory models we refer to Axsäter and Zipkin [35]. In the last decade, a stream of research that analyzes the so-called bullwhip effect has received attention. Lee et al. [22,23] introduced the term to describe the amplification of order variability along the supply chain. Chen et al. [6] quantified the impact of forecasting, lead times and information flows on the bullwhip effect.

Stochastic inventory models can be used to prove the optimality of inventory policies for certain settings, such as stationary or ARIMA demand. They can also be used to compute optimal order quantities and analyze order variability in these settings. However, stochastic inventory models can neither be used to