Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 176 (2007) 1723–1734
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Decision Support

A multiobjective evolutionary algorithm for approximating the efficient set

Thomas Hanne

Fraunhofer Institute for Industrial Mathematics (ITWM), Department of Optimization, Gottlieb-Daimler-Str. 49, Kaiserslautern 67663, Germany

Received 11 July 2002; accepted 25 May 2005
Available online 30 January 2006

Abstract

In this article, a new framework for evolutionary algorithms for approximating the efficient set of a multiobjective optimization (MOO) problem with continuous variables is presented. The algorithm is based on populations of variable size and exploits new elite preserving rules for selecting alternatives generated by mutation and recombination. Together with additional assumptions on the considered MOO problem and further specifications on the algorithm, theoretical results on the approximation quality such as convergence in probability and almost sure convergence are derived.

Keywords: Evolutionary computation; Multiple objective programming; Evolutionary algorithms; Continuous optimization; Efficient set; Stochastic convergence

1. Introduction

For multiobjective decision making (MODM) problems (see, e.g., Hanne, 2001a; Steuer, 1986; Vincke, 1992; Zeleny, 1982), a significant number of algorithms based on evolutionary approaches has been proposed during the last 15 years. Today, there are various survey articles of this research field available (see Fonseca and Fleming, 1995; Horn, 1997; Tamaki et al., 1996), specialized international conferences on evolutionary multi-criterion optimization take place (see the proceedings edited by Zitzler et al., 2001; Fonseca et al., 2003), and comprehensive monographs have been published (see Coello Coello et al., 2002; Deb, 2001). Theoretical results on evolutionary algorithms for multiobjective optimization such as, for instance, approximation proofs are, however, scarcely available. In this paper we introduce a new framework for evolutionary multiobjective algorithms which allows for an analysis of approximation.

E-mail address: hanne@itwm.fhg.de

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2005.05.031

Approximation, in that sense, means that, providing that sufficient computation time and memory is available, the algorithm is capable of reaching with an arbitrary exactness those alternatives which constitute in a mathematical sense the solution set which is usually denoted as the set of efficient or Pareto-optimal alternatives. The subsequent paper is organized as follows: In the next section some notation on multiobjective optimization problems is specified. In Section 3, an algorithm (or: algorithmic framework) capable of approximating the efficient set is introduced. In Section 4, further theoretical concepts for an analysis of approximation and additional assumptions are introduced and used for a corresponding proof. In Section 5, some conclusions are given.

2. Some notations

Originally, evolutionary algorithms (EAs) have been developed for scalar, or ordinary, optimization problems (see, e.g., Bäck et al., 1991, 1997), i.e. problems with a mathematical formulation as follows:

$$\max \quad f(x) \tag{2.1}$$

$$\text{s.t.} \quad x \in A \tag{2.2}$$

with $A \subseteq R^n$ called the feasible set and $f: R^n \to R$ being the objective function to be maximized. Instead of maximization a minimization can be assumed as well. The feasible set is usually defined by constraint functions,

$$A = \{x \in R^n : g_j(x) \leqslant 0, j \in \{1, \ldots, m\}\}. \tag{2.3}$$

Elements $a \in A$ are usually denoted as alternatives or (feasible) solutions. Traditional optimization methods (such as the simplex algorithm for linear optimization or Fletcher and Powell's (1963) method for nonlinear optimization) assume special properties for such optimization problems, e.g. the linearity of the objective function and the constraint functions, the convexity of the feasible set, or differentiability. In contrast to these methods, evolutionary algorithms are applicable to a wider range of optimization problems and are shown to be a robust and, at the same time, fast optimization method. In Schwefel's comparative study of search strategies for parameter optimization (cf. Schwefel, 1977, 1981, 1995) evolutionary algorithms performed comparatively fast (with respect to computation time) for a number of test problems while working more reliably on the average for the considered test problems.

These considerations also led to a generalization of EAs for optimization problems with several objective functions. Formally, such multiobjective optimization (MOO) problems can be described by

$$\text{“}\max\text{”} \quad f(x) \tag{2.4}$$

$$\text{s.t.} \quad x \in A, \tag{2.5}$$

where $f: R^n \to R^q$ is a vector-valued objective function and A is a feasible set defined as in (2.3). "max" means that each of the objective functions (the components of f) should be maximized. Usually, however, there does not exist a unique solution (if a solution exists) as in scalar optimization problems (2.1) and (2.2). Instead, mostly the set of efficient or Pareto-optimal alternatives is considered as the solution set of the problem (2.4) and (2.5). The Pareto or dominance relation "$\leqslant$" is defined by

$$x \leqslant y: \iff x_i \leqslant y_i \quad \forall i \in \{1, \ldots, q\} \text{ and } \exists i \in \{1, \ldots, q\} \text{ such that } x_i < y_i \tag{2.6}$$

for all $x, y \in R^q$. Using that relation, the set of efficient or Pareto-optimal alternatives is defined by

$$E(A, f) := \{x \in A : \nexists y \in A : f(x) \leqslant f(y)\}. \tag{2.7}$$

See Gal (1986) for a survey on specialized concepts of efficient sets.

The set of dominating alternatives with respect to a given set $B \subseteq R^n$ is defined as

$$\mathrm{Dom}(B, f) := \{x \in R^n : \exists y \in E(B, f) : f(y) \leqslant f(x)\}. \tag{2.8}$$

Usually, the efficient set contains several alternatives whereas in practice, decision makers desire a single alternative to be chosen as a final solution of a decision problem. Therefore, many MCDM methods utilize additional information like weights, achievement levels, trade-off information etc. (see Hanne, 2001a).