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Discrete optimization

The selective vehicle routing problem in a collaborative environment

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ABSTRACT

We consider a selective vehicle routing problem, in which customers belonging to different partners in a logistic coalition are served in a single logistic operation with multiple vehicles. Each partner determines a cost of non-delivery (CND) for each of its customers, and a central algorithm creates an operational plan, including the decision on which customers to serve and in which trip. The total transportation cost of the coalition is then divided back to the partners through a cost allocation mechanism.

This paper investigates the effect on the cost allocation of a partner's strategy on non-delivery penalties (high/low) and the properties of its customer locations (distance to the depot, degree of clustering). The effect of the cost allocation method used by the coalition is also investigated. We compare the well-known Shapley value cost allocation method to our novel problem-specific method: the CND-weighted cost allocation method.

We prove that an adequate cost allocation method can provide an incentive for each partner to behave in a way that benefits the coalition. Further, we develop a transformation that is able to transform any cost allocation into an individually rational one without losing this incentive.

1. Introduction and literature review

In recent years *horizontal collaboration* has become increasingly popular in the road transportation industry. The basic idea underlying this innovative business model is that distribution companies can significantly increase the efficiency of their operations by joining forces and becoming partners in a *horizontal logistic coalition*. Especially by solving a *collaborative vehicle routing problem*, i.e., a vehicle routing problem in which customers that would normally be served by different transportation companies are assigned to shared vehicle routes, less kilometres can be driven with trucks that have a higher average fill rate (Capgemini, 2008; Commission, 2011).

On the other hand, the added complexity of this novel way of working does not come without its challenges. One of the most important issues that needs to be tackled is that of *cost allocation* (also called *gain sharing*, depending on the perspective). A coalition incurs a single global coalition cost, which must be paid by the individual partners. The coalition must therefore install a method to allocate the total coalition cost to the partners. If a partner perceives its allocated share of the coalition cost to be too large, it might leave the coalition. Notwithstanding its importance, the cost allocation problem has been widely ignored in the literature on collaborative vehicle routing.

Specific contributions in the field of collaborative vehicle routing are still few and far between. The main body of research on this topic is focused on the demonstration of the gains by means of simulation (Blanc, Cruijssen, Fleuren, & De Koster, 2006; Cruijssen & Salomon, 2004; Ergun, Kuyzu, & Savelsbergh, 2007; Hageback & Segerstedt, 2004; Palander & Väätäinen, 2005), or by reporting on actual case studies (Bahrami, 2002; Cruijssen, Cools, & Dullaert, 2007; Defryn et al., 2014; Frisk, Göthe-Lundgren, Jörnsten, & Rönnqvist, 2010; Wiegmans, 2005). Studies on collaborative vehicle routing topics always aggregate the customers of the different partners into one single non-collaborative vehicle routing problem. In this way, however, company-specific strategies and objectives are ignored and the collaborating partners are implicitly merged into one entity. In this paper, we argue that solving a collaborative vehicle routing problem requires a more problem-specific approach, that explicitly takes into account the *interaction* between the vehicle routing problem and the cost allocation method. In Vanovermeire and Sörensen (2014a), an approach is developed that explicitly *integrates* the cost allocation method into the operational planning method, resulting in an optimization problem that looks for the least-cost solution under the constraints that each partner should be adequately rewarded for the changed delivery dates of its customers. Such an approach, however,

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considerably complicates the optimization problem and is therefore not a viable approach in all situations.

The Shapley value (Shapley, 1953), the Nucleolus (Leng & Parlar, 2005; Schmeidler, 1969), the Equal Profit Method (Frisk et al., 2010) and the volume-based allocation are some of the most well-known allocation methods. Some use a game theoretical approach (e.g., the Shapley value and the Nucleolus), others are based on simpler rules of thumb (e.g., the volume-based allocation and the Equal Profit Method).

As every allocation mechanism is based on a number of partner-specific characteristics (e.g., shipped volume, stand-alone cost, flexibility), choosing an allocation method results in an implicit selection of the desired partner behaviour. As an example, the volume-based allocation method allocates the profit of the coalition based on each partner's shipped volume and therefore implicitly stimulates partners to ship larger volumes. Stated differently, by agreeing on a certain cost allocation method, the partners implicitly or explicitly formulate a number of performance indicators they deem important for the coalition. Partners that behave well according to these predefined characteristics will be favoured by the cost allocation mechanism. This mechanism should therefore be used as an incentive for the partners to behave in favour of the coalition (Defryn et al., 2014). Dudek and Stadtler (2005) state that, by giving the right incentives, a solution can be obtained, that is optimal for the total coalition instead of a solution that is locally optimal for only one or a subset of partners.

There is widespread agreement in the literature that no single cost allocation method works best in all situations. In order to be able to include problem-specific elements into the allocation procedure, many authors therefore acknowledge the need for a case-specific approach (Biermasz, 2012; Defryn et al., 2014; Tijs & Driessen, 1986; Vanovermeire, Vercruysse, & Sörensen, 2014). The current literature, however, neglects the impact of the behaviour of an individual partner on the performance of the coalition. To guide this behaviour in a desirable direction, the coalition should give the right incentives to the partners, which, as mentioned, can be achieved by the appropriate cost allocation mechanism.

In this paper, we emphasize the *interaction* between these different elements — strategic partner behaviour, operational planning, and cost allocation — when operating in a collaborative environment. We focus on a relatively simple (yet realistic) collaborative variant of a well-known vehicle routing problem, the *selective vehicle routing problem*. This problem is formally described in Section 2. In Section 3 it is shown how this problem can be used in a collaborative environment. Here we focus on the issue of incorporating individual partner behaviour and a cost allocation method. By means of simulation, the properties and characteristics of the selective vehicle routing problem in a collaborative environment are analysed in Section 4. We highlight the notion of *bounded individual rationality* in Section 5. Finally, Section 6 summarises the main results and gives pointers for future research. All symbols used in this paper are summarised in Appendix A.

2. The selective vehicle routing problem

2.1. Problem definition and mathematical formulation

In the problem discussed in this paper, both the number of vehicles and the maximum distance each vehicle can travel, are limited. As a result, only a subset of customers can generally be served. The underlying operational problem is therefore a *selective vehicle routing problem (SVRP)*. In the vehicle routing literature, problems in which not all customers need to be visited, but a "reward" is gained for each customer visit are usually called *orienteering problems*, see e.g., Archetti, Hertz, and Speranza (2007); Bouly, Dang, and Moukrim (2010).

A formal description of the SVRP tackled in this paper is the following. We consider a set of c customers c_i ($i = \{1, \ldots, c\}$), with given coordinates in an euclidean distribution area, and a fixed fleet of v vehicles v_k ($k = \{1, \ldots, v\}$). The cost to travel between customers i and j is represented by the distance d_{ij}. Each vehicle can travel a predefined maximum distance D. Furthermore, a depot is given. Each vehicle starts and ends its distribution tour at this depot.

In the SVRP both the number of vehicles and the maximum distance travelled by each vehicle are limiting resources that may prevent all customers from being visited. A *compensation for non-delivery* (CND) is therefore determined for each customer. CND_i is the cost that is to be paid when customer i is not served, and may represent, e.g., a penalty paid to this customer in the form of a discount. We will elaborate on this concept in Section 3.1.

The aim of the SVRP is to determine a feasible subset of customers to be served, as well as the sequence in which these customers are visited by each vehicle in such a way that the *total distribution cost* is minimised. This cost includes both the total travel cost and the total CND value of all unvisited customers. The SVRP therefore implicitly assumes — without loss of generality — that travel distances and costs of non-delivery are expressed in the same units.

Formally we can define the SVRP as a mixed-integer programming problem. A complete list of symbols appears in Appendix A.

We use the subtour elimination constraints as defined by Vansteenwegen, Souffriau, and Oudheusden (2011). In this representation the position of customer i in the path of vehicle k is given by U_{ik}. Other decision variables are the following:

$$x_{ijk} = \begin{cases} 1 & \textit{if a visit to customer i is followed by a visit to} \\ & \quad \textit{customer j in the tour of vehicle k} \\ 0 & \textit{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1 & \textit{if customeriis served in the solution} \\ 0 & \textit{otherwise} \end{cases}$$

$$\min \left[\sum_{i=1}^{c} \sum_{j=1}^{c} \sum_{k=1}^{v} d_{ij} x_{ijk} + \sum_{i=1}^{c} (1 - y_i) \text{CND}_i \right] \quad (1)$$

Subject to

$$\sum_{i=1}^{c} x_{imk} = \sum_{j=1}^{c} x_{mjk} \qquad \forall m = 1 \ldots c, \forall k = 1 \ldots v \quad (2)$$

$$\sum_{k=1}^{v} \sum_{i=1}^{c} x_{ijk} = y_j \qquad \forall j = 1 \ldots c \quad (3)$$

$$\sum_{i=1}^{c} x_{0ik} = \sum_{j=1}^{c} x_{j0k} = 1 \qquad \forall k = 1 \ldots v \quad (4)$$

$$\sum_{i=0}^{c} \sum_{j=0}^{c} d_{ij} x_{ijk} \le D \qquad \forall k = 1 \ldots v \quad (5)$$

$$U_{ik} - U_{jk} + 1 \le (c - 1)(1 - x_{ijk}) \qquad \forall i, j = 1 \ldots c, \forall k = 1 \ldots v \quad (6)$$

$$1 \le U_{ik} \le c \qquad \forall i = 1 \ldots c, \forall k = 1 \ldots v \quad (7)$$

$$x_{ijk}, y_i \in \{0, 1\} \quad (8)$$

Constraints (2) ensure the connectivity of the path of a single vehicle, while Constraints (3) guarantee that every customer is visited at most once in the solution. Constraints (4) ensure that all vehicles start and end their trip at the depot (vertex 0). The maximal allowed vehicle distance is ensured by constraints (5). Constraints (6) and (7) take care of the subtour elimination.