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Dynamic speculation and hedging in commodity futures markets with a stochastic convenience yield

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ABSTRACT

The main objective of this paper is to address, in an a continuous-time framework, the issue of using storable commodity futures as vehicles for hedging purposes when, in particular, the convenience yield as well as the market prices of risk evolve randomly over time. Following the martingale route and by operating a suitable constant relative risk aversion utility function (CRRA) specific change of numéraire, we solve the investor's dynamic optimization program to obtain quasi analytical solutions for optimal demands, which can be expressed in terms of two discount bonds (traded and synthetic). Contrary to the existing literature, we explicitly derive the individual optimal proportions invested in the spot commodity, in a discount bond and in the futures contracts, which can be computed in a simple recursive way. We suggest various decompositions allowing an investor to assess the sensitivity of the optimal demands to the state variables and to specify the role played by each risky asset. Empirical evidence shows that the convenience yield has a strong impact on the speculation and hedging positions and the interaction among time-varying risk premia determines the magnitude and the sign of these positions.

1. Introduction

Futures markets have experienced a dramatic growth, worldwide, of both trading volume and contracts written on a wide range of underlying assets. These features make it easier to use futures contracts as hedging instruments against unfavorable changes in the investment opportunity set, i.e. changes in state variables describing the economic/financial environment. The growing activity of these markets has been accompanied, since the original normal backwardation (Hicks, 1939; Keynes, 1930) and the theory of storage (Brennan, 1958; Kaldor, 1939; Working, 1949), by a substantial body of literature devoted to pricing and hedging with futures contracts. Especially, investments in commodity futures have, in recent years, risen significantly. Indeed, commodities are considered by fund managers as an alternative asset class to traditional assets such as stocks and bonds for two main reasons: they may improve (stocks and bonds) portfolio diversification benefits, and may be efficient hedging instruments against the inflation risk. However, investors are also exposed to commodities risk. Although, a growing theoretical and empirical literature examines these two aspects (see, for example, Andriosopoulos & Nomikos, 2014; Bae, Kim, & Mulvey, 2014; Bodie, 1983; Dai, 2009; Daskalaki & Skiadopoulos, 2011; Erb & Harvey, 2006; Geman & Kharoubi, 2008; Gorton & Rouwenhorst, 2006; Kat & Oomen, 2007), there is no paper, in an intertemporal framework, dealing with the question of how to hedge commodities risk. The main objective of this paper is to address, in a continuous-time context, the issue of using storable commodity futures, by an unconstrained investor, as vehicles for speculative and hedging purposes.

The recent high fluctuations in commodity prices have revived the interest in commodity risk management through essentially futures contracts. The convenience yield, in accordance with the theory of storage, turns out to be the crucial variable, which constitutes one of the main differences between commodity prices and prices of financial assets. Surprisingly, while there are a number of models dealing with futures hedging, to the best of our knowledge, the specific case of commodity futures contracts with a stochastic convenience yield has not yet been addressed in the relevant literature. However, it is widely recognized that it evolves randomly over time. Moreover, a growing number of empirical studies on commodity return predictability stress its important role (see, Besembinder & Chan, 1992; Fama & French, 1987; Hong & Yogo, 2010; Khan, Khokher,

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& Simin, 2007). In addition, the evidence of predictability is consistent with time varying risk premia in commodities. In our environment, to the extent that spot prices, futures prices and inventory decisions are related (see, for example, Brennan, 1958; Litzenberger & Rabinowitz, 1995; Routledge, Seppi, & Spatt, 2000), we would expect market prices of risk to be stochastic. This is in line with some papers studying asset allocation with time varying prices of risk (see, for instance, Kim & Omberg, 1996; Lioui & Poncet, 2001; Liu, 2007; Munk & Sorensen, 2004; Sangvinatsos & Wachter, 2005; Wachter, 2002).

This paper provides a model of optimal demand that could better account for the way both the stochastic convenience yield and stochastic (affine) market prices of risk affect the optimal demand of an unconstrained investor[2]. In order to do so, following the reference models in the literature (Casassus & Collin-Dufresne, 2005; Hilliard & Reis, 1998; Schwartz, 1997), the economic framework retains the spot commodity price, the instantaneous interest rate and the convenience yield as the relevant imperfectly correlated mean-reverting state variables associated with the dynamics of the futures price. The optimal demand for commodity futures contracts is derived for an investor who maximizes the expected constant relative risk aversion (CRRA) utility function of her (his) lifetime consumption and final wealth by following the no-arbitrage martingale approach (Cox & Huang, 1989; Karatzas, Lehoczky, & Shreve, 1987). This framework takes into account the main characteristics of commodity markets and provides explicit solutions up to resolution of ordinary differential equation (ODEs) (see Liu, 2007) for the optimal demand, which is classically composed of a speculative part and of a hedging term.

We estimate the parameters of the model by using weekly data on U.S. Treasury bills and on West Texas Intermediate (WTI) light sweet crude oil futures contracts traded on ICE (Intercontinental Exchange) for the period 2001–2010. Since the spot price, the short rate, and the convenience yield are not directly observable, the estimation is based on the Kalman filter method, which is the appropriate method when state variables are not observable and are Markovian (see, for instance, Manoliu & Tompaidis, 2002; Schwartz, 1997; Trolle & Schwartz, 2009). The results show mean reversion in the short rate and the convenience yield. They also reveal that risk premia are time varying amplifying mean reversion in these variables. The spot price is positively correlated with the convenience yield (theory of storage), while, as Frankel and Hardouvelis (1985) have suggested, it is negatively correlated with the short rate.

A thorough study of the speculative and of the hedging components allows us to enrich the analysis of optimal demands by going beyond the existing studies by suggesting various decompositions. Usually, in a continuous-time framework, papers obtain general formulae for these two components without deriving specific formulae for each risky asset. In the case of futures contracts for a constrained investor, Adler and Detemple (1988a, 1988b) suggested expressions for the futures contract and the spot. We generalize this result along the lines of our framework by deducing the individual speculative and hedging proportions invested in the spot commodity, a discount bond and the futures contract, which may be computed in a useful recursive way underlying the interactions between risky assets demands and by taking into account some commodity markets features. In particular, empirical estimations reveal that mean-reversion in the state variables and in the prices of risk as well as the correlation between the assets determine the sign of the speculative and hedging positions.

As a consequence of the calculation of the individual proportions for each asset, our analysis clarifies the role played by the primitive assets and the futures contract when speculating and hedging. Our analysis also calls into question Breeden (1984) result according to which the primitive assets are ineffective in hedging the risk of the state variables. Indeed, it assigns primitive assets and the futures contract a specific task: hedging the risk of the state variables.

An important question is to know how state variables affect optimal demands. In other words, what are the implications of the predictive variables on optimal demands? In our model, the investor is able to assess the sensitivity of the optimal demand, through the sensitivity of an investor-specific bond (synthetic), to the state variables and can therefore rule on the relevance of the investment opportunity set. Estimation shows that the convenience yield has a strong effect on the speculative and on the hedging proportions.

The remainder of the paper is organized as follows. In Section 2, the economic framework is described and the investor's optimization problem is formulated. Section 3 is devoted to the derivation of the optimal asset allocation for the unconstrained investor. The estimation of the parameters of the model and a discussion of the behavior of optimal demands, based on those estimations, are given in Section 4. Section 5 offers some concluding remarks and suggests some potential future extensions.

2. The general economic framework

Consider a continuous-time frictionless economy. The uncertainty in the economy is represented by a complete probability space (Ω, F, P) with a standard filtration $F = \{F_t : t \in [0, T]\}$, a finite time period [0, T] with $T > 0$, the historical probability measure P and a 3-dimensional vector of independent standard Brownian motions, $z(t)' = (z_S(t), z_u(t), z_v(t))$, defined on (Ω, F), where $'$ stands for the transpose.

In this section, following Hilliard and Reis (1998), Schwartz (1997) and Casassus and Collin-Dufresne (2005), three imperfectly correlated state variables, represented by the vector $Y(t)' = [X(t)\ r(t)\ \delta(t)]'$, are assumed to be associated with the dynamics of the futures prices: the logarithm of spot commodity price, $X(t) = Ln(S(t))$, the instantaneous riskless interest rate, $r(t)$, and the instantaneous convenience yield, $\delta(t)$. In the sequel of the paper, $\lambda_i(.)$ and σ_i stand for the market prices of risk related to the variables and the strictly positive instantaneous volatility of the variables respectively, while ρ_{ij}, with $i \neq j$, denotes the correlation coefficient for $i = X(t), r(t), \delta(t)$. Σ_{kl}, with $k \neq l$ represents either the covariance between the assets or between the assets and the variables.

$X(t)$ satisfies the following stochastic differential equation (SDE hereafter):

$$dX(t) = \left(r(t) - \delta(t) + \sigma_S\lambda_X(X(t), r(t), \delta(t)) - \frac{1}{2}\sigma_S^2\right)dt + \sigma_S dz_S(t) \tag{1}$$

with initial condition $LnS(0) \equiv LnS$.

The short rate is governed by a mean-reverting process as in Vasicek (1977):

$$dr(t) = \alpha(\vartheta - r(t))dt + \sigma_r\left[\rho_{sr}dz_S(t) + \rho_{ur}dz_u(t)\right] \tag{2}$$

with initial condition $r(0) \equiv r$. $\rho_{ur} = \sqrt{1-\rho_{Sr}^2}$. The short rate has a tendency to revert to a constant long-run interest rate level, ϑ, with a constant speed of mean reversion α.

The instantaneous convenience yield evolves stochastically over time by following a mean-reverting process:

$$d\delta(t) = k\left(\bar{\delta} - \delta(t)\right)dt + \sigma_\delta\left[\rho_{S\delta}dz_S(t) + \rho_{u\delta}dz_u(t) + \rho_{v\delta}dz_v(t)\right] \tag{3}$$

with initial condition $\delta(0) \equiv \delta$. The convenience yield has a tendency to revert to a constant long-run convenience yield, $\bar{\delta}$, with a constant speed of mean-reversion k. Empirical studies (see Fama & French, 1988; Brennan, 1991) found that the convenience yield should be

[2] Other models examining dynamic asset allocation with futures (see, among others, Adler and Detemple, 1988a, b; Duffie and Jackson, 1990; Briys et al., 1990; Duffie and Richardson, 1991; Lioui et al., 1996) deal with a constraint investor.