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An efficient algorithm for capacitated assortment planning with stochastic demand and substitution

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ABSTRACT

The ever increasing product variety in grocery retailing is in conflict with the limited shelf space. Managing the assortment and regular order quantity for perishable and non-perishable products is therefore a core task for retailers. Retailers want to maximize profit by considering potential revenues, purchase costs, diminishing profits for products and opportunity costs for unfulfilled demand when determining assortment sizes and order volume to meet stochastic consumer demand.

We considered a newsvendor-based decision model that jointly optimizes assortment size and the associated order volumes for listed items. The model considers out-of-assortment (OOA) and out-of-stock (OOS) substitution effects. The model is a structural equivalent to the model described in Kök and Fisher (2007) and solved there via a heuristic procedure. We develop an optimal solution procedure and a time-efficient heuristic that solves the model. The heuristic suggested is applicable for practically relevant, large-scale problem instances. Numerical tests reveal that the heuristic procedure produces close-to-optimal solutions and outperforms the Kök and Fisher heuristic with regard to both computational time and solution quality. A simulation study with discrete demand justifies our approach with using continuous demand distributions. The numerical analyses show that considering substitution effects in a decision-based model has a significant impact on the total profit and solution structure. Further managerial insights are presented with regard to assortment heterogeneity when shelf size, substitution and demand variability are varied.

1. Introduction

Several mutually reinforcing trends have made efficient assortment planning one of the most critical marketing and operational decisions for retailers (Mantrala et al., 2009). Retailers and consumer good producers rate "optimization of the product portfolio and category management" the key task for achieving performance goals (Breuer, Fritsch, Prauschke, & Steegmann, 2009). Product availability is one of top three factors affecting customer satisfaction (Eltze, Goergens, & Loury, 2013). These results are not surprising as competition for shelf space in retail stores is at an all-time high, driven by the competitive need to constantly introduce new items. The average number of items in overall store assortments has increased by 30 percent since 2000 (EHI Retail Institute, 2014). In confectionery, for example, the number of brands rose by more than 40 percent between 1997 and 2001, but overall sales by only 0.8 percent (Carlotti, Coe, & Perrey, 2006). Assortment levels have become so excessive that reducing variety greatly increases sales (Boatwright & Nunes, 2001; Sloot & Verhoef, 2008). It needs to be kept in mind that the increasing online market is leading to new customer shopping behavior that may also influence assortment decisions (Hübner, Holzapfel, & Kuhn, 2015). Even if retailers could determine the optimal assortment mix for each individual customer, it may be unprofitable to stock such an assortment. Out-of-shelf situations are therefore inevitable. Empirical studies show that between 45 percent and 84 percent of demand can be substituted (Aastrup & Kotzab, 2009; Campo, Gijsbrechts, & Nisol, 2003; Gruen, Corsten, & Bharadwaj, 2002; Xin, Messinger, & Li, 2009). These consumer reactions need to be reflected in assortment optimization. Tan and Karabati (2013) and Karabati, Tan, and Öztürk (2009) show the impact of substitution on service levels and how point-of-sales data can be used to estimate substitution rates. In addition, Kuhn and Sternbeck (2013) conducted an exploratory study with 28 European retailers showing that operational planning questions are impacted by assortment decisions.

As a result, retailers need to manage the complexity of satisfying consumer demand cost efficiently by determining the interdependent problems of assortment size and shelf inventory. In these

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circumstances they have to match consumer demand with shelf supply by balancing variety (number of products) and shelf service levels (number of items of a product). Offering broader assortments thus limits appropriate service levels and vice versa, as shelf space is scarce. Consequently, retailers need to make same-time decisions on which products to offer, how much space to allocate and how much to reorder for each product. This is a tactical decision problem as it results in mid-term changes, e.g., in supplier selection and shelf layout and therefore belongs to category planning (Hübner, Kuhn, & Sternbeck, 2013a; 2013b). The decision problem receives input from strategic planning and informs operational instore fulfillment tasks (Hübner & Kuhn, 2014). It entails three main aspects: (i) Assortment planning involves listing products by including the most profitable items according to consumer demand. Total demand consists of initial demand for a product and also the substitution by and complementary demand for other products. (ii) Shelf space planning assigns quantities under the constraints of limited shelf sizes. (iii) Instore inventory planning determines restocking policies. This is concerned with holding as little inventory as possible to ensure efficient refill costs and limit the risk that items will perish while still meeting demand, i.e., not getting out of stock. Replenishment frequency impacts the entire shelf supply and actual stock levels, which then influence consumer choice at the shelves. These planning problems are interrelated. For example, if a retailer lists more brands, he has less space for each brand (given that shelf space is limited) and needs to reorder the items more frequently, which reduces storage duration but increases the risk of getting out of stock. The problem is especially important for perishable items with a limited shelf life, but also applicable to non-perishable items. An assortment model needs to balance supply and demand effects. Firstly, assortment decisions with limited shelf space affect possible out-of-assortment (OOA) or out-of-stock (OOS) demand substitutions for unavailable items when the shopper enters the store. Secondly, inventory decisions such as space allocation and associated reorder volume impact operational refill costs and service levels, but also have to take into account space limitations. Finally, retailers need to choose their assortment and order volume carefully as consumer demand is stochastic and perishable items cannot be stored or can only be sold at a discount later whereas non-perishable items than impose higher storage costs. The demand can also not be backlogged.

Retailers typically solve decision problems sequentially: determining the assortment, allocating it to the shelf and finally determining order sizes. They tend to rely on software-based deterministic decision support. However these solutions use intuitive heuristics for the assortment decision problem (Hübner & Kuhn, 2012). Using heuristics is intuitive and often simple to understand, but does not result in optimal assortment configurations, especially when demand is stochastic. The available scientific models, however, are also limited concerning optimality. Our literature review shows that they are also limited to a very narrow set of items that can be optimized. A comprehensive and efficient decision model will therefore improve decision making and profit (Hübner & Kuhn, 2012). We propose applying a stochastic newsvendor model for perishable and non-perishable items that jointly optimizes assortment and order sizes under storage constraints, and enables the efficient solution of retail-specific problem sizes using an optimal procedure or—if problem instances are too large—a near-optimal solution heuristic procedure. We assume product categories that are jointly delivered once per sales period, e.g., once per day, every second day, once per season, etc. (see also Holzapfel, Hübner, Kuhn and Sternbeck, 2015, Sternbeck & Kuhn, 2014). Product examples for these categories with limited sale periods are fresh products (e.g., fruits, vegetables, bakery, meat, and dairy products), seasonal products (e.g., fashion, consumer electronics, seasonal fruits, holiday products), promotional products (e.g., "buy one, get one free", product bundles, short-term price offers) or temporary offers (e.g., additional assortments for limited sales periods, pop-up stores, seasonal markets, promotions at the gondola end and in the promotion area). In addition, we assume that the stores have no or only very limited backroom storage available for these categories. This is because of their location in city centers, shopping malls, or the need for special facilities (e.g., refrigeration).

The rest of this paper is organized as follows. We first provide an overview of the associated literature in Section 2, and then develop the model in Section 3. In Section 4 we elaborate appropriate solution approaches to solve the problem. Section 5 presents computational tests. Finally, Section 6 discusses potential areas for future research.

2. Literature review

We will investigate the related literature in three streams. The first stream deals with deterministic shelf space management models, while multi-item newsvendor problems are investigated in another related stream. Finally, we analyze newsvendor-based multi-item assortment problems with substitution effects.

Integrative assortment planning models have their origin in shelf space management that assumes a deterministic demand. Common denominators of these models are item demand as a function of the space allocated to an item and limited shelf space. The continuous inventory review model by Urban (1998) takes into account inventory-elastic demand. Yücel, Karaesmen, Salman, and Türkay (2009) develop three different models to examine the effects of substitution between products, supplier selection and shelf space limitations on the optimal assortment decision. The first model does not consider substitution effects, the second does not allow retailers to select their suppliers, and the third model implies unlimited shelf space. However, Yücel et al. (2009) prove that shelf space has a significant impact on the performance of assortments. Irion, Lu, Al-Khayyal, and Tsao (2012) use a piecewise linearization for modeling space- and cross-space elasticity. The logarithmic function, however, does not take into account effects of OOA substitution. The main drawback of all these studies is their limited applicability under conditions of stochastic demand. The deterministic models assume efficient instore logistics so that shelves are always stocked and consumer demand is deterministic. These assumptions can be valid to some extent for non-perishable products, but do not hold for perishable items.

Secondly, the related problem for perishable products is often described by the newsvendor model. The basic stochastic one-period one-item model is well known in literature (see for example Turken et al. (2012)). Extending this to include multi-item newsvendor problems has also been widely studied in literature, see for example reviews by Khouja (1999), Qin, Wang, Vakharia, Chen, and Seref (2011), and Turken et al. (2012). Multi-item newsvendor models with one constraint have been studied by Lau and Lau (1996). The latter develop solution procedures for strictly positive demand functions as well as demand functions with long tails, and extend their approach to a multi-constraint scenario. Abdel-Malek and Montanari (2005) extend the approach of Lau and Lau (1996) to ensure positive optimal order quantities. They formulate thresholds to estimate the tightness of the constraint. Abdel-Malek, Montanari, and Morales (2004) solve the newsvendor model under a budget constraint with multiple demand distributions. Lau and Lau (1995) apply multiple constraints in their newsvendor model. However, all these models do not take assortment planning and substitution effects into account.

Finally, the following assortment models use the newsvendor as a basic model to find the profit maximizing shelf configuration. The main feature of assortment planning is the integration of consumers' willingness to accept a substitute when their favorite product is not available. An up-to-date literature review on planning models that considers product substitution is presented by Shin, Park, Lee, and Benton (2015).

The most popular approaches for estimating demand substitution in assortment planning are *multinomial logit models* (MNL) and *ex-*