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Decision Support

Strategic decentralization in binary choice composite congestion games

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ABSTRACT

This paper studies strategic decentralization in binary choice composite network congestion games. A player decentralizes if she lets some autonomous agents to decide respectively how to send different parts of her stock from the origin to the destination. This paper shows that, with convex, strictly increasing and differentiable arc cost functions, an atomic splittable player always has an optimal unilateral decentralization strategy. Besides, unilateral decentralization gives her the same advantage as being the leader in a Stackelberg congestion game. Finally, unilateral decentralization of an atomic player has a negative impact on the social cost and on the costs of the other players at the equilibrium of the congestion game.

1. Introduction

This paper introduces strategic decentralization into composite network congestion games, and studies its properties in a specific subclass of such games. A player decentralizes her decision-making if she lets each of her deputies decide independently how to send the part of her stock deputed to him from its origin to its destination. A unilateral decentralization can be beneficial or deleterious for the decentralizing player herself, and it also has an influence on the other players' utility and the social welfare as well. This paper provides a detailed analysis of these problems in the case where all the players have the same binary choice.

In a network congestion game, i.e. routing game, each player has a certain quantity of stock and a finite set of choices. A choice is a directed, acyclic path from the player's origin to her destination. A player with a stock of infinitesimal weight is *nonatomic*. She has to attribute her stock to only one choice. A player with a stock of strictly positive weight is *atomic*. She (more rigorously, her stock) is *splittable* if she can divide it into several parts and affect each part to a different choice. She can also be non splittable, which is the case originally studied in the seminal work of Rosenthal (1973) on congestion games. This paper considers only the splittable case so that the word *splittable* is often omitted. A path is composed of a series of arcs, and the cost of a path is the sum of the costs of its component arcs. The cost entailed to a user of an arc depends on the total weight of the stocks on it as well as on the quantity of that user's stock on it. A player wishes to minimize her cost, which is the total cost to her stock. A game with both nonatomic and atomic players is called a *composite* game. An equilibrium in a composite congestion game is called a *composite equilibrium* (CE for short) (Boulogne, Altman, Kameda, & Pourtallier, 2002; Harker, 1988; Wan, 2012; Yang & Zhang, 2008). An equilibrium does not necessarily minimize the social cost, i.e. the total cost to all the players.

In a composite congestion game, an atomic player of weight m decentralizes if she is replaced by a composite set of players called her *deputies* (i.e. n atomic players of weight $\alpha^1, \dots, \alpha^n$ and a set of nonatomic players of total weight α^0, such that $\sum_{i=0}^{n} \alpha^i = m$) who have the same choice set as her, and she collects the sum of her deputies' costs as her own.

Here is an example of advantageous decentralization. Two atomic players both have a stock of weight $\frac{1}{2}$ to send from O to D. Two parallel arcs link O to D, with per-unit cost function $c_1(t) = t + 10$ and $c_2(t) = 10t + 1$ respectively. At the equilibrium, both players send weight $\frac{2}{11}$ on the first arc and $\frac{7}{22}$ on the second one. The cost is 4.14 to both players and the social cost is 8.28. If player 1 deputes her stock to two atomic deputies both of weight $\frac{1}{4}$, then at the equilibrium of the resulting congestion game, both deputies send weight $\frac{1}{44}$ on the first arc, while player 2 sends weight $\frac{1}{4}$ there. The cost is 2.06 to both deputies of player 1. Hence player 1 gains by decentralizing because her current cost 4.12 is lower than 4.14. However, player 2's cost is now 4.59 and the social cost is 8.71, both higher than before.

Assuming that the arc cost functions are convex, strictly increasing and continuously differentiable in congestion, this paper obtains the following properties of unilateral decentralization in composite congestion games with binary choice or, equivalently, in a two-terminal two-parallel-arc composite routing game:

(i) For the atomic player who decentralizes unilaterally, all her decentralization strategies are weakly dominated by

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single-atomic ones which depute her stock to at most one atomic deputies in addition to nonatomic ones (Theorem 2.1). A fortiori, she possesses an optimal decentralization strategy (Theorem 3.1), which depends on her relative size among all the players.
(ii) Unilateral decentralization gives an atomic player the same advantage as being the leader in a Stackelberg congestion game (Theorem 3.2).
(iii) After the unilateral decentralization of an atomic player, the social cost at the equilibrium increases or does not change, and the cost to each of her opponents increases or does not change (Theorem 4.1).

Although the above results are obtained in the specific settingf of binary choice games, the goal of this paper is to introduce the notion of strategic decentralization into composite congestion games, to point out its significance, and to initiate a systematic study of its properties.

The paper is organized as follows. Section 2 presents the model, defines decentralization, and shows the special role of single-atomic decentralization strategies. Section 3 proves the existence of an optimal unilateral decentralization strategy, and shows that unilateral decentralization gives an atomic player the same advantage as being the leader in a Stackelberg congestion game. Section 4 focuses on the impact of unilateral decentralization on the social cost and the other players' cost. Section 5 concludes. The proofs and auxiliary results are regrouped in Section 6.

1.1. Related literature

The "inverse" concept of decentralization – coalition formation or collusion between players – has been extensively studied. Hayrapetyan, Tardos, and Wexler (2006) first define the *price of collusion* (PoC) of a parallel network to be the ratio between the worst equilibrium social cost after the nonatomic players form disjoint coalitions and the worst equilibrium social cost without coalitions. Bhaskar, Fleischer, Huang, Eisenbrand, and Shepherd (2010) extended this study to series-parallel networks. (A series-parallel network can be constructed by merging in series or in parallel several graphs of parallel arcs.) This index is closely related to another important notion: the *price of anarchy* (PoA), which is introduced by Koutsoupias, Papadimitriou, Meinel, and Tison (1999) (and Papadimitriou, 2001) as the ratio between the worst equilibrium social cost and the minimal social cost in nonatomic games. Cominetti, Correa, and Stier-Moses (2009) derive the first bounds on the PoA with atomic players. For a specific network structure, one can deduce the PoC by the PoA with atomic players and the PoA with nonatomic players. Further results on the bound of the PoA with atomic players are obtained in Harks (2011), Roughgarden and Schoppmann (2011) and Bhaskar et al. (2010). Roughgarden and Tardos (2002) and Correa, Schulz, and Stier-Moses (2008) provide fundamental results on the bound of the PoA with nonatomic players. PoA in nonatomic games with asymmetric costs or elastic demands is studied in Perakis (2007) and Chau and Sim (2003), among others.

Beyond the coalitions formed by nonatomic players, Cominetti et al. (2009), Altman et al. (2011), and Huang (2013) consider those formed by atomic players. Their results can be interpreted as the impact of certain kinds of collusion and hence, the "inverse" of it, decentralization, on the social cost. Wan (2012) studies the impact of coalition formation on the nonatomic players' cost outside the coalition in parallel-link networks. In terms of the impact of coalition formation on the cost of the coalition members themselves, Cominetti et al. (2009), Altman et al. (2011) and Wan (2012) provide examples in different contexts of disadvantageous coalition formation for the members themselves. These are actually examples of advantageous decentralization. Finally, for works on strategic decentralization, one can cite Sorin and Wan (2013) in integer-splitting congestion games, and Baye, Crocker, and Ju (1996) in industrial organization (where they call the strategic decentralization of a firm "divisionalization").

Finally, let us point out that the above-mentioned coalition formation is studied by the approach of comparative statics in a non-cooperative game setting. It is different from the cooperative routing games studied in Quant, Borm, and Reijnierse (2006) and Blocq and Orda (2014).

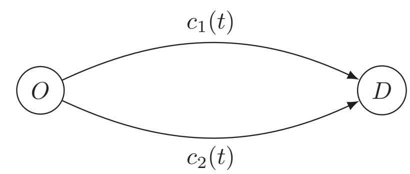

Fig. 1. A binary choice congestion game.

2. Model and preliminary results

2.1. Binary choice composite congestion games

In Fig. 1, nodes O and D are linked by two parallel arcs. The per-unit cost function of arc r is c_r, for $r = 1, 2$. When the total weight of stocks on arc r is t, the cost to each unit of them is $c_r(t)$. Both c_1 and c_2 are defined on Ω, a neighborhood of $[0, \bar{M}]$, with $\bar{M} > 0$. They satisfy the following assumption throughout this paper.

Assumption 1. Both c_1 and c_2 are strictly increasing, convex and continuously differentiable on Ω, and non-negative on $[0, \bar{M}]$.

There is a continuum of nonatomic players of total weight T^0, and N atomic players of strictly positive weight $T^1, T^2, \ldots, T^N$ respectively, where $N \in \mathbb{N}$. If there are no nonatomic (resp. no atomic) players, then $T^0 = 0$ (resp. $N = 0$). Let $I = \{0, 1, \ldots, N\}$. The player profile is denoted by $T = (T^i)_{i \in I}$, and their total weight is $M = \sum_{i \in I} T^i$, with $M < \bar{M}$.

The profile of the nonatomic players' strategies is described by their flow $x^0 = (x^0_1, x^0_2)$, where x^0_r is the total weight of the nonatomic players on arc r. The strategy of atomic player i is specified by her flow $x^i = (x^i_1, x^i_2)$, where x^i_r is the weight that she sends by arc r. Call $x = (x^i)_{i \in I}$ the (system) flow. Denote respectively by $X^i = \{x^i \in \mathbb{R}^2_+ \mid x^i_1 + x^i_2 = T^i\}$ the space of feasible flows for the nonatomic players or an atomic player i, and by $X = \prod_{i \in I} X^i$ the space of feasible system flows. Let $\xi = (\xi_r)_{r \in \{1,2\}}$ be a vector function defined on X by $\xi_r(x) = \sum_{i \in I} x^i_r$, i.e. the aggregate weight on arc r. For $i \in I$, let $x^{-i} = (x^j)_{j \in I \setminus \{i\}}$.

With flow x, the cost to a nonatomic player taking arc r is $c_r(\xi_r(x))$. The cost to atomic player i is $u^i(x) = x^i_1 c_1(\xi_1(x)) + x^i_2 c_2(\xi_2(x))$. The social cost is $CS(x) = \xi_1(x) c_1(\xi_1(x)) + \xi_2(x) c_2(\xi_2(x))$.

Let this composite congestion game be denoted by $\Gamma(T)$. Flow $x \in X$ is a *composite equilibrium* (CE) of $\Gamma(T)$ if (Harker, 1988):

(a) for $r \in \{1, 2\}$, if $x^0_r > 0$, then $c_r(\xi_r(x)) \le c_s(\xi_s(x))$ for all $s \in \{1, 2\}$; and
(b) for $i \in I \setminus \{0\}$, x^i minimizes $u^i(\cdot, x^{-i})$ on X^i.

Like all composite congestion games taking place in a two-terminal parallel-arc networks, game $\Gamma(T)$ always admits a unique CE. The reader is referred to Richman and Shimkin (2007) or Wan (2012) for a proof. For the uniqueness of equilibria in congestion games with different types of players and in more general networks, see, for example, Meunier and Pradeau (2014) Milchtaich (2005) Richman and Shimkin (2007) and (Bhaskar, Fleischer, Hoy, & Huang, 2015). Let the nonatomic players' common cost at the unique CE x be denoted by $u^0(x)$.