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Decision Support

Frontier projection and efficiency decomposition in two-stage processes with slacks-based measures

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ABSTRACT

In the prior literature on measuring the efficiency of two-stage processes, there are both radial and non-radial methods of efficiency measurement. In some cases, non-radial methods which allow all inputs, intermediate measures and outputs to change non-proportionally are more appropriate than radial methods, but they do not ensure stage efficiency or allow for the efficiency decomposition of two-stage processes. Based on slacks-based measure (SBM), this paper develops both envelopment-based and multiplier-based models to obtain simultaneously both the frontier projection and the efficiency decomposition. Specifically, we propose the variable intermediate measures SBM (VSBM) model to evaluate the system efficiency of two-stage processes and consider the following three properties of the VSBM model: 1) we derive the efficient DEA frontier projection based on the VSBM model; 2) we address potential conflicts in this model with respect to the intermediate measures; 3) we prove that the system inefficiency is equivalent to the sum of inefficiencies of the two stages. Furthermore, we derive the efficiency decomposition of two-stage processes based on the dual of the VSBM model. Finally, we apply our proposed approach to real data of US commercial banks, and extend our approach to settings in which the assumption of variable returns to scale (VRS) holds or there are more general network structures.

1. Introduction

Based on the fundamental contributions by Debreu (1951), Shephard (1953), and Farrell (1957), Charnes, Cooper, and Rhodes (1978) proposed the well-known performance evaluation method entitled *data envelopment analysis* (DEA) to measure the relative efficiency of peer decision making units (DMUs). In traditional DEA models, most notably the CCR model (Charnes et al., 1978) and its later reformulations, DMUs are treated as black boxes whose internal structure is ignored. A major drawback of this is that inefficiencies arising from the organization of activities within this internal structure cannot be addressed, which limits the amount information that can be gained to improve the system efficiency.

In recent years, DEA has been extended to investigate the performance of two-stage processes or network structures. Unlike traditional DEA studies which treat production as a one-stage process, the analysis of these two-stage processes or network structures makes it possible to examine the internal structure of DMUs by explicitly modelling the two stages. In the simplest two-stage structure, all outputs from the first stage are viewed as intermediate measures (and/or products) which constitute the inputs to the second stage (see Fig. 1 below). Many real-world examples of such two-stage processes have been reported. For example, Wang, Gopal, and Zionts (1997) examined the impact of information technology (IT) on firm performance within the two-stage DEA framework. They focused on IT-related value-added activity and identified the effects of IT on intermediate output variables, which in turn affected the performance of 22 banks whose value-added activities consisted of the processes of (i) collecting and (ii) investing funds. Based on Chen and Zhu (2004), Wang et al. (1997) and Rho and An (2007) measured the indirect impact of IT on firm performance by identifying the efficient frontier of two principal value-added stages and by extending two-stage DEA models to consider input and output slacks. Seiford and Zhu (1999) used DEA to examine the performance of US commercial banks by employing a basic two-stage process of which the first stage was profitability and the second was marketability. For more details, we refer to Cook, Liang, and Zhu (2010) and Kao (2014a).

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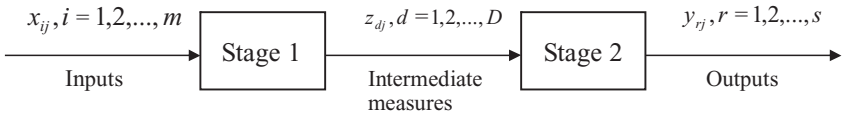

Fig. 1. Two-stage processes.

Based on the special structure of two-stage processes, several models have been proposed based on radial efficiency measures. Typically, these models are developed based on the arithmetic or geometric mean efficiency decomposition which makes it possible to determine the efficiency of each stage, also known as divisional efficiency (Chen, Cook, Kao, & Zhu, 2013). For example, Chen, Du, Sherman, and Zhu (2010), Du, Liang, Chen, Cook, and Zhu (2011), Kao and Hwang (2008), Liang, Cook, and Zhu (2008), Zha and Liang (2010) and Li, Chen, Liang, and Xie (2012) defined the overall efficiency of two-stage processes as the product of efficiencies of each stage. Chen and Zhu (2004), Chen, Cook, Li, and Zhu (2009) and Cook, Zhu, Bi, and Yang (2010), on the other hand, defined the overall efficiency as the weighted sum of divisional efficiencies. In this paper, we call the divisional efficiency as stage efficiency when we refer to the two-stage process in Fig. 1. This term is clearer because the production with respect to the two-stage process proceeds stage by stage.

Tone and Tsutsui (2009) argued that one should be careful when using radial DEA models to measure the efficiency of two-stage processes, because radial efficiency measures assume that all inputs or outputs change proportionally. As a partial remedy, they introduced a slacks-based measure (SBM) to develop a network DEA approach to evaluate both divisional and overall efficiencies of DMUs. Other extensions have been proposed by Fukuyama and Weber (2010), Tone and Tsutsui (2010, 2014) and Kao (2014b). In particular, Tone and Tsutsui (2010, 2014) extended the SBM-based network DEA to the dynamic DEA. Based on Fukuyama and Weber (2010), Tone and Tsutsui (2009) measured the eco-efficiencies of two-stage processes which generated some harmful (undesirable) outputs. Kao (2014b) studied the efficiency decomposition in network DEA based on SBM.

However, Chen et al. (2013) showed that Tone and Tsutsui's (2009) approach may not fulfil the property of stage efficiency, suggesting that the rationale for the (stage) efficiency for two-stage processes must be reconsidered. Moreover, the establishment of the DEA frontier or DEA projection is a key issue for determining the reference set for inefficient DMUs and for improving their performance. Even though Tone and Tsutsui's (2009) approach can produce efficient projected DMUs, the problem of frontier projection also needs to be studied to ensure stage efficiency. Recently, Chen et al. (2013) claimed that the multiplier-based network DEA model should be used for determining the stage efficiency while the envelopment-based network DEA model should be used for determining the frontier projection for inefficient DMUs. Lim and Zhu (2016) showed that the duality in the standard radial DEA migrated to the two-stage network DEA well.

This paper develops both envelopment-based and multiplier-based models to derive the frontier projection for inefficient DMUs and to measure simultaneously both stage and system efficiencies of two-stage processes. In particular, we propose a SBM-based DEA model, called the *variable intermediate measures SBM model* (VSBM), to measure system efficiency. We prove that the proposed model gives an efficient frontier projection. We also show that the proposed model helps address potential conflicts between the two stages. Specifically, we find that the same weights are associated with the intermediate measures in the two stages when the VSBM model is transformed into its multiplier (dual) model. We also find that the system inefficiency is equivalent to the sum of inefficiencies of the two stages. Finally, we propose several multiplier-based models to establish an efficiency decomposition (the stage efficiency in two-stage processes) based on the dual of the VSBM model.

The remainder of this paper is organized as follows. Section 2 reviews the SBM model. We then propose a SBM-based model for two-stage processes to calculate the system efficiency in Section 3. In Section 4, we present key properties of the VSBM model and discuss the efficiency decomposition of two-stage processes. In Section 5, we apply the proposed approach to a data set of US commercial banks. Section 6 extends our approach to the assumption of variable returns to scale (VRS) and general network structures. Conclusions and directions for future research are given in Section 7.

2. The slacks-based measure model

Traditional CCR model uses radial measure of technical efficiency which assumes that all inputs and outputs can change proportionally. Tone (2001) extended this model to non-radial measures by proposing the slacks-based measure (SBM) model. This model employs slacks to model the excessive use of inputs and the insufficient production of outputs relative to efficient DMUs. It is a non-oriented DEA model which differs from traditional (input-oriented or output-oriented) DEA models.

Assume that there are n DMUs with inputs $x_{ij}, i = 1, 2, \ldots, m$ and outputs $y_{rj}, r = 1, 2, \ldots, s$. Then, the SBM efficiency of DMU_0 is given by:

$$\begin{aligned}
\min \quad & \rho_0 = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{i0}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{r0}}} \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} = x_{i0} - s_i^-, \quad i = 1, 2, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} = y_{r0} + s_r^+, r = 1, 2, \ldots, s, \\
& \lambda_j \geq 0, j = 1, 2, \ldots, n, \\
& s_i^-, s_r^+ \geq 0, \quad i = 1, 2, \ldots, m; r = 1, 2, \ldots, s.
\end{aligned} \tag{1}$$

where λ_j is the intensity variable, and s_i^- and s_r^+ are input and output slacks, respectively. The variable λ_j indicates the importance of the j-th DMUs in constructing the efficient frontier for DMU_0 under evaluation. The slacks s_i^- and s_r^+ may be different for different inputs and outputs so that inputs and outputs can change non-proportionally in the computation of efficiencies. The last constraint in model (1) ensures that inputs can only decrease and outputs can only increase.

Note that model (1) can be regarded as a general case of the CCR model. Specifically, if $s_i^- = s^-, s_r^+ = 0, \forall i, r$ or $s_i^- = 0, s_r^+ = s^+, \forall i, r$, then the model (1) can be transformed into the input-oriented or output-oriented CCR models, respectively. Denote the optimal value of model (1) as ρ_0^*. According to Tone (2001), we have the following definition:

Definition 1. (*SBM-efficient*). DMU_0 is SBM-efficient if and only if $\rho_0^* = 1$.

3. The variable intermediate measures SBM model

Consider the two-stage production process in Fig. 1. The first stage uses m inputs $x_{ij}, i = 1, 2, \ldots, m$ to produce D intermediate measures (regarded as outputs) $z_{dj}, d = 1, 2, \ldots, D$. The second stage treats these intermediate measures as inputs to generate s (final) outputs $y_{rj}, r = 1, 2, \ldots, s$.