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Decision Support

Hesitant analytic hierarchy process

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ABSTRACT

In traditional analytic hierarch process (AHP), decision makers (DMs) are required to provide crisp judgments over paired comparisons of objectives to construct comparison matrices. To enhance the modeling ability of traditional AHP, we propose hesitant AHP (H-AHP) that can consider the hesitancy experienced by the DMs in decision. H-AHP is characterized by hesitant judgments, where each hesitant judgment can be represented by several possible values. Different probability distributions can be used to further describe hesitant judgments according to the DMs' preferences. Based on a hesitant comparison matrix (HCM) that consists of hesitant judgments, we define two indices to measure the consistency degree and the consensus degree of the HCM respectively. From a stochastic point of view, a new prioritization method is developed to derive priorities from HCMs, where the results are with probability interpretations. We provide a step by step procedure for H-AHP, and demonstrate this new method with a real-life decision making problem.

1. Introduction

Analytic hierarchy process (AHP) can be used for structuring, measurement, and synthesis. It has wide applications, such as the decisions of choice, prioritization/evaluation, resource allocation, benchmarking, quality management, public policy, and forecasting etc. (Saaty, 1989, 1994, 1977, 1980, 2008; Saaty & Vargas, 1987). In a multi-criteria environment, AHP is built on a decision maker (DM)'s intrinsic ability to structure his/her perceptions or his/her ideas hierarchically. With the produced dimensionless ratio-scale priorities, AHP assists the DMs to make reliable decisions.

To resolve the AHP problems, four main steps should be followed. (1) Modeling: determining a top-to-bottom form as a hierarch with different levels of criteria, sub-criteria and alternatives. (2) Evaluation: constructing comparison matrices based on a 1–9 scale. (3) Prioritization: using prioritization methods to derive local priorities of objectives in each level of the hierarchy. (4) Synthesis: utilizing aggregation procedures to synthesize the local priorities into global priorities of alternatives.

In AHP, the DMs usually provide crisp values for judgments over paired comparisons of objectives with respect to a criterion. So we call the judgments represented by crisp values as crisp judgments. If the DMs are uncertain about the judgments, this uncertainty can be measured by intervals which can be called interval judgments (Saaty & Vargas, 1987). Both crisp judgments and interval judgments are based on the 1–9 scale. If it is difficult to determine crisp or interval values, the judgments can also be represented by fuzzy values (Ishizaka & Labib, 2011; Ishizaka & Nguyen, 2013; Saaty, 1980; Van Laarhoven & Pedrycz, 1983; Zadeh, 1965).

However, the existing representations of judgments still have limitations in practice. More possible representations should be developed to represent uncertainty experienced by the DMs due to the increasing complexity of modern society. Motivated by a real-life AHP problem, that is to assess the strategic positions of islands and reefs of China, we need to deal with a hesitant case, where the military experts prefer to retain several possible values to represent some judgments rather than crisp or interval values. For example, to make the judgment over a paired comparison of objectives, a military expert is hesitant about the values 4 or 5 based on the 1–9 scale. So we have two possible values in this case, not a single value represented by a crisp judgment, or a margin of error represented by an interval judgment. From a probability point of view, 4 and 5 should be with the same probability 0.5. Similar idea of this kind of uncertainty can also be found in the concept of hesitant fuzzy sets proposed by Torra (2010).

In the process of human reasoning and concept formation, hesitancy is a common phenomenon, especially in decision making. We use several possible values to indicate a judgment in AHP to describe the hesitancy experienced by the DMs in decision. As distinct from crisp judgments and interval judgments, we call the judgment represented by several possible values a hesitant judgment. Based on

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Table 1
The fundamental 1–9 scale.

Intensity of importance	Definition	Explanation
1	Equal importance	Two objectives contribute equally to the objective.
3	Moderate importance	Experience and judgment slightly favor one objective over another.
5	Strong importance	Experience and judgment strongly favor one objective over another.
7	Very strong or demonstrated importance	An objective is favored very strongly over another, its dominance demonstrated in practice.
9	Extreme importance	The evidence favoring one objective over another is of the highest possible order of affirmation.
2,4,6,8	For compromise between the above values	Sometimes one needs to interpolate a compromise judgment numerically because there is no good word to describe it.
Reciprocals of above	If objective *i* has one of the above nonzero numbers assigned to it when compared with objective *j*, then *j* has the reciprocal value when compared with *i*	A comparison mandated by choosing the smaller objective as the unit to estimate the larger one as a multiple of that unit
1.1–1.9	For tied objectives	When objectives are close and nearly indistinguishable; moderate is 1.1 and extreme is 1.9

hesitant judgments and under the framework of AHP, we propose a hesitant AHP (H-AHP) in this paper with a series of new concepts and methods.

The rest of this paper is organized as follows. Section 2 gives necessary descriptions of AHP. Section 3 develops the concept of hesitant comparison matrices (HCMs), then focuses on consistency and consensus of HCMs respectively. In Section 4, we develop hesitant preference analysis as a new prioritization method for HCMs. Section 5 gives related discussions. Based on the developed concepts and methods above, Section 6 concludes a step by step procedure for H-AHP. A real-life example is introduced to illustrate our results in Section 7. Section 8 closes this paper with some conclusions.

2. Descriptions of analytic hierarchy process

In recent decades, AHP and related research topics have widely attracted much attention of researchers (Ivlev, Vacek, & Kneppo, 2015; Kułakowski, 2015; Siraj, Mikhailov, & Keane, 2015; Zhu, Xu, Zhang, & Hong, 2015). In this section, we discuss four main steps of analytic hierarchy process (AHP) in detail, which are modeling, evaluation, prioritization and synthesis to provide a fundamental for our new method.

2.1. Modeling and evaluation

Modeling is the first step of AHP, that is to hierarchically structure a problem. With respect to a decision goal or control criterion, the hierarchy consists of several levels with criteria, sub-criteria and alternatives from top to bottom. As usual, we begin with the alternatives, then go up with the simplest sub-criteria until determining all objectives in each level.

In the evaluation step of AHP, the decision makers (DMs) provide judgments over paired comparisons of objectives with respect to a criterion in an upper level. Saaty (1990) gave the 1–9 scale shown in Table 1, which is a scale of absolute numbers used to assign numerical values to judgments made by comparing two objectives.

The judgments over paired comparisons of objectives are collected by comparison matrices. Let $A = (a_{ij})_{n\times n}$ be a comparison matrix, it can be shown as follows:

$$A = \begin{bmatrix} 1 & a_{12} & a_{13} & \dots & a_{1n} \\ & 1 & a_{23} & \dots & a_{2n} \\ & \vdots & 1 & & \vdots \\ \vdots & 1/a_{ij} & \vdots & \ddots & \vdots \\ & \dots & & & 1 \end{bmatrix} \quad (1)$$

In the evaluation step of hesitant AHP (H-AHP), the DMs are allowed to provide hesitant judgments to construct hesitant comparison matrices (HCMs), which are discussed in Section 3 in detail.

Table 2
The average *RI* for different sizes of *A*.

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

2.2. Consistency

Consistency is a basic requirement for comparison matrices to guarantee meaningful results, and consistency check and consistency improving are two main research topics.

2.2.1. Two consistency indices

Checking consistency of comparison matrices is a crucial step to avoid misleading solutions. Saaty (1977) developed an eigenvector method (EVM) to derive priorities from comparison matrices, and then defined a consistency index to measure their consistency degrees.

For a set of objectives $X = \{x_1, x_2, \dots, x_n\}$, and a constructed comparison matrix $A = (a_{ij})_{n\times n}$, the EVM is based on solving the equation:

$$A\omega = \lambda_{\max}\omega, \ \sum_{i=1}^{n} \omega_i = 1 \quad (2)$$

where $\lambda_{\max}$ is the maximum eigenvalue of A, and ω is the priority vector of the objectives.

The consistency index is defined as $CR = CI/RI$, where $CI = (\lambda_{\max} - n)/(n-1)$, and RI is a random consistency index. For different sizes of A, the average RI is shown in Table 2 (Saaty, 1977).

If $CR < 0.1$, it means that the error in measurement is considered to be acceptable, then A is said to be with the acceptable consistency; otherwise, A is called unacceptable.

Besides the EVM that derives priorities from comparison matrices, another popular method is a row geometric mean method (RGMM) developed by Crawford and Williams (1985). For the comparison matrix A, and according to the RGMM, the priorities ω_i $(i = 1, 2, \dots, n)$ can be simply found as the geometric means of the rows of A:

$$\omega_i = \frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}\right)^{\frac{1}{n}}} \quad (3)$$

Based on the priorities ω_i $(i = 1, 2, \dots, n)$, Aguaron and Moreno-Jiménez (2003) developed a geometric consistency index (GCI) to measure consistency of A:

$$GCI_A = \frac{2}{(n-1)(n-2)} \sum_{i<j} \log^2 e_{ij} \quad (4)$$

where $e_{ij} = a_{ij}\omega_j/\omega_i$.