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Decision Support

Fuzzy set approach to the utility, preference relations, and aggregation operators

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Abstract

Score $\mathbf{x} = (x_1, \ldots, x_n)$ describing an alternative α is modelled by means of a continuous quasi-convex fuzzy quantity $\mu_\alpha = \mu_\mathbf{x}$, thus allowing to compare alternatives (scores) by means of fuzzy ordering (comparison) methods. Applying some defuzzification method leads to the introduction of operators acting on scores. A special stress is put on the Mean of Maxima defuzzification method allowing to introduce several averaging aggregation operators. Moreover, our approach allows to introduce weights into above mentioned aggregation, even in the non-anonymous (non-symmetric) case. Finally, Ordered Weighted Aggregation Operators (OWAO) are introduced, generalizing the standard OWA operators.

Keywords: Decision support systems; Fuzzy sets; Aggregation operator; Defuzzification; Dissimilarity; Fuzzy quantity; Fuzzy utility; Preference relation

1. Introduction

The aim of this contribution is a proposal of a fuzzy set based approach to the utility function and the preference relation choice. Observe that the early fuzzy set based approaches to the decision making are summarized in the monograph of Fodor and Roubens [7]. Recent results in this domain are collected in [3]. The paper is organized as follows. In the next section, a general approach to the fuzzy utility functions

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based on dissimilarity functions is introduced. Section 3 is devoted to a proposal of a weak preference relation derived from the fuzzy utility function by means of some ordering of fuzzy quantities, including the orderings based on defuzzification methods. In the Section 4, the MOM (mean of maxima) method is applied to derive a relevant non-decreasing value function (idempotent aggregation operator on the corresponding score). Finally, OWAO (Ordered Weighted Aggregation Operators) are introduced.

2. Fuzzy utility function

In several decision making models, the choice of an appropriate alternative is transformed to the problem of comparison of some available quantitative information. Recall, e.g., the standard optimization problems arising from the minimal costs or the maximal profit, several ordering approaches such as leximin or discrimin, see [6], etc. The simplest situation occurs when each alternative is described by a single real value (say the costs), in which case from two alternatives we choose the cheaper one. From mathematical point of view, we exploit here the standard ordering on the real line. Much more complex is the situation when alternatives are described by fuzzy reals [5,10]. In that case, there are several orderings known so far. For an exhaustive overview we recommend [18,19]. Because of greater flexibility and modelling power, we focus our attention to the last case, i.e., to the decision problems for alternatives characterized by fuzzy quantities.

We recall first some basic notations from the fuzzy sets theory [20,4]. For a given non-empty set $\Omega \neq \emptyset$ (universe), a fuzzy set V (fuzzy subset V of Ω) is characterized by the membership function $\mu_V\colon \Omega \to [0,1]$. In this context, classical sets (subset of Ω) are called crisp sets (crisp subsets of Ω) and they are characterized by the corresponding characteristic function. The height $\mathrm{hgt}(V)$ of the fuzzy set V is given by

$$\mathrm{hgt}(V) = \sup_{\omega\in\Omega} \mu_V(\omega).$$

Evidently, for crisp set V it holds $\mathrm{hgt}(V) = 1$ whenever V is non-empty. For a given constant $d \in [0,1]$, the corresponding d-cut $V^{(d)}$ of a fuzzy set V is given by

$$V^{(d)} = \{\omega \in \Omega | \mu_V(\omega) \geqslant d\}.$$

Fuzzy set V is called normal if $V^{(1)} \neq \emptyset$, i.e., $V(\omega) = 1$ for some $\omega \in \Omega$. Fuzzy subsets of a real interval I are called fuzzy quantities. Moreover, a fuzzy quantity V is called *quasi-convex* whenever each d-cut $V^{(d)}$ is a convex subset of I, i.e., $V^{(d)}$ is an interval for all $d \in [0,1]$. Equivalently, quasi-convexity of a fuzzy quantity V can be characterized by the fulfillment of inequality

$$\mu_V(\lambda r + (1-\lambda)s) \geqslant \min(\mu_V(r), \mu_V(s)) \tag{1}$$

for all $r, s \in I$ and $\lambda \in [0,1]$. For more details see [5,10].

On the set of alternatives $\mathscr{A}$, let each alternative α be described by the score $\mathbf{x} = (x_1, \ldots, x_n)$, where n is the number of applied criteria and $x_1, \ldots, x_n \in I$ are the single score from some prescribed real interval I (usually, $I = [0,1]$ or $I = \mathbb{R}$). In the criterion i, the dissimilarity $D_i(x,y)$ of a score x and another score y, with $x, y \in I$, is described by the dissimilarity function $D_i : I^2 \to \mathbb{R}$, such that

$$D_i(x,y) = K_i(f_i(x) - f_i(y)),$$

where $K_i : \mathbb{R} \to \mathbb{R}$ is a convex function with the unique minimum $K_i(0) = 0$ (shape function), and $f_i : I \to \mathbb{R}$ is a strictly monotone continuous function (scale transformation). Evidently, each D_i is then continuous. Observe that this approach to dissimilarity is based on the ideas of verbal fuzzy quantities as proposed and discussed in [11–13]. Note also that the concept of dissimilarity functions is closely related to the penalty functions proposed by Yager and Rybalov [16], compare also [2]. Finally, remark that the dissimilarity function D is related to some standard metric on the interval I whenever it is symmetric, i.e., if K is an even