Available online at www.sciencedirect.com

SCIENCE DIRECT®

European Journal of Operational Research 169 (2006) 742–750

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/ejor

A heuristic for minimizing the expected makespan in two-machine flow shops with consistent coefficients of variation

Pawel Jan Kalczynski, Jerzy Kamburowski *

Department of Information Operations and Technology Management, College of Business Administration,
The University of Toledo, Toledo, OH 43606-3390, USA

Received 12 May 2003; accepted 26 August 2004
Available online 14 March 2005

Abstract

The paper deals with the classical problem of minimizing the makespan in a two-machine flow shop. When the job processing times are deterministic, the optimal job sequence can be determined by applying Johnson's rule. When they are independent and exponential random variables, Talwar's rule yields a job sequence that minimizes the makespan stochastically.

Assuming that the job processing times are independently and Weibull distributed random variables, we present a new job sequencing rule that includes both Johnson's and Talwar's rules as special cases. The proposed rule is applicable as a heuristic whenever the job processing times are characterized by their means and the same coefficient of variation. Simulation results show that it leads to very encouraging results when the expected makespan is minimized.

Keywords: Stochastic scheduling; Flow shop; Expected makespan; Weibull distribution; Coefficient of variation; Heuristic

1. Introduction

A set of jobs, $\{1, 2, \ldots, n\}$, available at time zero has to be processed in a shop with $m = 2$ machines A and B. Each job is processed first on A and next on B. No machine can process more than one job at a time, no job preemption is allowed, all setup times are included in the job processing times, and there is unlimited intermediate storage between the machines. The problem is to determine a job sequence (permutation) that minimizes the completion time of the last job, also known as the makespan. Recall that the problem is strongly

---

[*] Corresponding author. Tel.: +1 419 530 4361; fax: +1 419 530 2290.
E-mail address: jerzy.kamburowski@utoledo.edu (J. Kamburowski).

0377-2217/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2004.08.045

NP-hard for $m \geqslant 3$ [6], and permutation schedules are dominant for $m = 2$ and $m = 3$ [10], that is, an optimal job sequence is then an optimal schedule among non-permutation schedules.

Assume that a_k and b_k denote the deterministic processing times of job k on machines A and B, respectively. In a breakthrough paper on scheduling theory, Johnson [10] proposed to apply the rule:

$$\text{job } i \text{ precedes job } j \text{ if } \min(a_j, b_i) > \min(a_i, b_j). \tag{1}$$

He showed that it yields a transitive ordering among the jobs, and every job sequence satisfying (1) has the minimum makespan.

Assume now that A_k and B_k are independent and exponential random variables representing the processing times of job k on machine A and B. Let a_k and b_k denote the means of A_k and B_k. Talwar [17] conjectured that the sequence minimizing the expected makespan can be determined by applying the rule:

$$\text{job } i \text{ precedes job } j \text{ if } E\big(\min(A_j, B_i)\big) = \frac{a_j b_i}{a_j + b_i} > \frac{a_i b_j}{a_i + b_j} = E\big(\min(A_i, B_j)\big), \tag{2a}$$

that is equivalent to,

$$\text{job } i \text{ precedes job } j \text{ if } \frac{1}{a_i} - \frac{1}{b_i} > \frac{1}{a_j} - \frac{1}{b_j}. \tag{2b}$$

Cunningham and Dutta [3] were the first to prove Talwar's conjecture, and observe that the permutation schedules are dominant when the expected makespan is minimized. Ku and Niu [12] showed later that Talwar's sequence also minimizes the makespan stochastically, and the permutation schedules remain then dominant. (A random variable X is smaller than another random variable Y in the sense of the stochastic order, written $X \leqslant_{\mathrm{st}} Y$, if $P(X > t) \leqslant P(Y > t)$ for every t.) They also presented a sufficient condition for two adjacent job interchange from which both rules (1) and (2) can be derived.

Let A_k and B_k be independent Weibull distributed random variables with means a_k and b_k, and a common shape parameter c. For such job processing times we propose the following sequencing rule:

$$\text{job } i \text{ precedes job } j \text{ if } E\big(\min(A_j, B_i)\big) = \frac{a_j b_i}{\left(a_j^c + b_i^c\right)^{1/c}} > \frac{a_i b_j}{\left(a_i^c + b_j^c\right)^{1/c}} = E\big(\min(A_i, B_j)\big), \tag{3a}$$

that is equivalent to,

$$\text{job } i \text{ precedes job } j \text{ if } \frac{1}{a_i^c} - \frac{1}{b_i^c} > \frac{1}{a_j^c} - \frac{1}{b_j^c}. \tag{3b}$$

Our extensive simulation experiments, in which we employed the *Ranlux* pseudo random number generator [8], led us to the conjecture that the rule (3) might minimize the expected makespan.

For $c = 1$, the Weibull distribution with mean μ and shape parameter c becomes the exponential distribution with mean μ. When c approaches infinity, it reduces to the degenerate distribution at μ. Since (3) reduces to (1) for $c = \infty$, and to (3) for $c = 1$, our sequencing rule (3) is a generalization of both Johnson's and Talwar's rules.

A constant parameter c for all job processing time distributions can be regarded as a constant coefficient of variation. Therefore, whenever the inequality $E(\min(A_j, B_i)) > E(\min(A_i, B_j))$ does not induce a transitive ordering among the jobs for arbitrary job processing time distributions characterized by their means and the same coefficient of variation, we propose to apply (3) with Weibull's shape parameter c computed for this coefficient. We claim that such a heuristic leads to very encouraging results when the expected makespan is minimized.

Excellent overviews of rather limited research on stochastic scheduling are presented in Pinedo [13], and in Chapter 13 of [16] written by Righter. Using simulation, Dodin [4] examined, in particular, the performance of Johnson's rule applied to different job processing time distributions. Similar studies were conducted by Portugal and Trietsch [14]. Elmaghraby and Thoney [5] proposed a heuristic for the expected makespan minimization that is based upon Johnson's sequence modifications. Assuming that the job processing time distributions are unknown but bounded, Allahverdi and Sotskov [1] examined the existence of a unique optimal sequence.