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A modified standard embedding with jumps in nonlinear optimization ☆

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Abstract

The paper deals with a combination of pathfollowing methods (embedding approach) and feasible descent direction methods (so-called jumps) for solving a nonlinear optimization problem with equality and inequality constraints. Since the method that we propose here uses jumps from one connected component to another one, more than one connected component of the solution set of the corresponding one-parametric problem can be followed numerically. It is assumed that the problem under consideration belongs to a generic subset which was introduced by Jongen, Jonker and Twilt. There already exist methods of this type for which each starting point of a jump has to be an endpoint of a branch of local minimizers. In this paper the authors propose a new method by allowing a larger set of starting points for the jumps which can be constructed at bifurcation and turning points of the solution set. The topological properties of those cases where the method is not successful are analyzed and the role of constraint qualifications in this context is discussed. Furthermore, this new method is applied to a so-called modified standard embedding which is a particular construction without equality constraints. Finally, an algorithmic version of this new method as well as computational results are presented.

Keywords: Parametric programming; Pathfollowing methods with jumps; Genericity; Jongen–Jonker–Twilt regularity; Modified standard embedding

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1. Introduction

Let $\mathbb{R}^n$ be the n-dimensional space with the Euclidean norm $\|\cdot\|$ and $C^k(\mathbb{R}^n, \mathbb{R})$, $k \geqslant 1$ the space of k-times continuously differentiable functions. In this paper we consider the nonlinear optimization problem

$$(P) \quad \min\{f(x) \mid x \in M\}, \tag{1.1}$$

where the non-empty feasible set is defined by finitely many equality and inequality constraints as

$$M = \{x \in \mathbb{R}^n \mid h_i(x) = 0,\ i \in I,\ g_j(x) \leqslant 0,\ j \in J\},$$

with $I = \{1,\ldots,m\}$, $m < n$, $J = \{1,\ldots,s\}$, and $f, h_i, g_j \in C^3(\mathbb{R}^n, \mathbb{R})$, $i \in I, j \in J$. Furthermore, we introduce the one-parametric nonlinear optimization problem

$$P(t) \quad \min\{f(x,t) \mid x \in M(t)\}, \tag{1.2}$$

where $t \in \mathbb{R}$ is a real parameter,

$$M(t) = \{x \in \mathbb{R}^n | h_i(x,t) = 0,\ i \in I,\ g_j(x,t) \leqslant 0,\ j \in J\}$$

and $f, h_i, g_j \in C^3(\mathbb{R}^n \times \mathbb{R}, \mathbb{R})$, $i \in I, j \in J$. For sake of simplicity we assume that all functions in (1.1) and (1.2) are three times continuously differentiable although some of the results given here also hold for a lower degree of differentiability.

The *embedding approach* is a well-known method for the calculation of a solution point (local minimizer, stationary point, generalized critical point, etc.) of (P); its basic idea is to construct an auxiliary problem $P(t)$ which satisfies at least the following three conditions:

(A1) A solution point x^0 of $P(0)$ is known.
(A2) The set of solution points of $P(t)$ is non-empty for all $t \in [0,1]$.
(A3) $P(1)$ is similar (in a certain sense) or equivalent with (P).

Then, by using a so-called pathfollowing (or homotopy or continuation) method a solution point x^* of the original problem (P) can be obtained by following numerically a solution path connecting $(x^0, 0)$ and $(x^*, 1)$, i.e. one has to find a discretization

$$0 = t_0 < \cdots < t_i < \cdots < t_N = 1$$

of the interval $[0, 1]$ and corresponding solution points $x(t_i)$ of $P(t_i)$, $i = 0,\ldots,N$ (cf. e.g. [1–5,7,8,10–12,16,18,19,21,23]).

Example 1.1. As an example we present the so-called *standard embedding* which is defined by the one-parametric problem

$$P_{x^0}(t) \min\{tf(x) + (1-t)\|x - x^0\|^2 \mid x \in M_{x^0}(t)\},$$

with the starting point $x^0 \in \mathbb{R}^n$ and the feasible set

$$M_{x^0}(t) = \left\{ x \in \mathbb{R}^n \,\middle|\, \begin{array}{l} h_i(x) + (t-1)h_i(x^0) = 0,\ i \in I \\ g_j(x) + (t-1)g_j(x^0) \leqslant 0,\ j \in J \end{array} \right\}.$$

Obviously, (A1) and (A3) are satisfied but (A2) cannot be guaranteed in general (cf. Example 5.1). In particular, the feasible set could be empty for some parameter values $t \in (0, 1)$.

However, in general, the existence of a solution curve to be followed is a very strong condition. In [16,18], topological conditions are discussed which ensure an appropriate structure of the solution set of