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Invited Review

Thirty years of heterogeneous vehicle routing

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ABSTRACT

It has been around 30 years since the heterogeneous vehicle routing problem was introduced, and significant progress has since been made on this problem and its variants. The aim of this survey paper is to classify and review the literature on heterogeneous vehicle routing problems. The paper also presents a comparative analysis of the metaheuristic algorithms that have been proposed for these problems.

1. Introduction

In the classical Vehicle Routing Problem (VRP) introduced by Dantzig and Ramser (1959), the aim is to determine an optimal routing plan for a fleet of homogeneous vehicles to serve a set of customers, such that each vehicle route starts and ends at the depot, each customer is visited once by one vehicle, and some side constraints are satisfied. There exists a rich literature on the VRP and its variants, see, e.g., the surveys by Cordeau, Laporte, Savelsbergh, Vigo, Barnhart, and Laporte (2007) and Laporte (2009), and the books by Golden, Raghavan, and Wasil (2008) and Toth and Vigo (2014).

In most practical distribution problems, customer demands are served by means of a heterogeneous fleet of vehicles (see, e.g., Hoff, Andersson, Christiansen, Hasle, & Løkketangen, 2010; FedEx, 2015; TNT, 2015). Fleet dimensioning or composition is a common problem in industry and the trade-off between owning and keeping a fleet and subcontracting transportation is a challenging decision for companies. Fleet dimensioning decisions predominantly involve choosing the number and types of vehicles to be used, where the latter choice is often characterized by vehicle capacities. These decisions are affected by several market variables such as transportation rates, transportation costs and expected demand.

The extension of the VRP in which one must additionally decide on the fleet composition is known as the Heterogeneous Vehicle Routing Problem (HVRP). HVRPs are rooted in the seminal paper of Golden, Assad, Levy, and Gheysens (1984) published some 30 years ago and have recently evolved into a rich research area. There have also been several classifications of the associated literature from different perspectives. Baldacci, Battarra, and Vigo (2008) provided a general overview of papers with a particular focus on lower bounding techniques and heuristics. The authors also compared the performance of existing heuristics described until 2008 on benchmark instances. Baldacci, Toth, and Vigo (2010a) presented a review of exact algorithms and a comparison of their computational performance on the capacitated VRP and HVRPs, while Hoff et al. (2010) reviewed several industrial aspects of combined fleet composition and routing in maritime and road-based transportation. More recently, Irnich, Schneider, and Vigo (2014) briefly reviewed papers on HVRPs published from 2008 to 2014.

This paper makes three main contributions. The first is to classify heterogeneous vehicle routing problems. The second is to present a comprehensive and up-to-date review of the existing studies. The third is to comparatively analyze the performance of the state-of-the-art metaheuristic algorithms. Our review differs from the previous ones by including references that have appeared since 2008, by comparing heuristic algorithms, and by including industrial applications and case studies.

The remainder of this paper is structured as follows. The HVRPs and its variants are described and classified in Section 2. Extended reviews of the three main problem types, namely the Fleet Size and Mix Vehicle Routing Problem, the Heterogeneous Fixed Fleet Vehicle Routing Problem and the Fleet Size and Mix Vehicle Routing Problem with Time Windows are presented in Sections 3–5, respectively. Reviews of the other variants, extensions and case studies are presented in Sections 6 and 7. A tabulated summary of the literature and

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comparisons of the state-of-the-art heuristic algorithms are provided in Section 8. The paper closes with some concluding remarks and future research directions in Section 9.

2. Classification of the heterogeneous vehicle routing problem

We first define and classify the variants of HVRPs in Section 2.1, and then present three mathematical formulations in Section 2.2.

2.1. Problem definition and classification

HVRPs generally consider a limited or an unlimited fleet of capacitated vehicles, where each vehicle has a fixed cost, in order to serve a set of customers with known demands. These problems consist of determining the fleet composition and vehicle routes, such that the classical VRP constraints are satisfied. Two major HVRPs are the *Fleet Size and Mix Vehicle Routing Problem* (FSM[1]) introduced by Golden et al. (1984) which works with an unlimited heterogeneous fleet, and the *Heterogeneous Fixed Fleet Vehicle Routing Problem* (HF) introduced by Taillard (1999) in which the fleet is predetermined. Other variants of the FSM and the HF also exist. In what follows, we will classify the main variants with respect to two criteria: (i) objectives and (ii) presence or absence of time window constraints. We will also mention other HVRP variants and extensions.

2.1.1. Objectives

The objective of both the FSM and the HF is to minimize a total cost function which includes fixed (F) and variable (V) vehicle costs. We now differentiate between five important variants: 1) the FSM with fixed and variable vehicle costs, denoted by FSM(F, V), introduced by Ferland and Michelon (1988); 2) the FSM with fixed vehicle costs only, denoted FSM(F), introduced by Golden et al. (1984); 3) the FSM with variable vehicle costs only, denoted by FSM(V), introduced by Taillard (1999); 4) the HF with fixed and variable vehicle costs, denoted by HF(F, V), introduced by Li, Golden, and Wasil (2007); 5) the HF with variable vehicle costs only, denoted by HF(V), introduced by Taillard (1999).

2.1.2. Time windows

Two natural extensions of the FSM and HF arise when time window constraints are imposed on the start of service at each customer location. These problems are denoted by FSMTW and HFTW, respectively. In these extensions, two measures are used to compute the total cost to be minimized: 1) The first is based on the en-route time (T) which is the sum of the fixed vehicle cost and the trip duration but excludes the service time. In this case, service times are used only to check route feasibility and for performing adjustments to the departure time from the depot in order to minimize pre-service waiting times; 2) The second cost measure is based on distance (D) and consists of the fixed vehicle cost and the distance traveled by the vehicle, as is the case in the standard VRP with Time Windows (VRPTW) (Solomon, 1987).

The FSM and HF, combined with the two objectives above, give rise to four problem types: 1) the FSMTW with objective T, denoted by FSMTW(T), introduced by Liu and Shen (1999b); 2) the FSMTW with objective D, denoted by FSMTW(D), introduced by Bräysy, Dullaert, Hasle, Mester, and Gendreau (2008); 3) the HFTW with objective T, denoted by HFTW(T), introduced by Paraskevopoulos, Repoussis, Tarantilis, Ioannou, and Prastacos (2008); 4) the HFTW with objective D, denoted by HFTW(D), recently introduced by Koç, Bektaş, Jabali, and Laporte (2015).

2.1.3. Other variants

More involved variants of the FSM or of the HF have been defined, including those with multiple depots (see Dondo & Cerdá, 2007; Bettinelli, Ceselli, & Righini, 2011, 2014). Other extensions include stochastic demand (Teodorović, Krčmar-Nozić, & Pavković, 1995), pickups and deliveries (Irnich, 2000; Qu & Bard, 2014), multi-trips (Prins, 2002; Seixas & Mendes, 2013), the use of external carriers (Chu, 2005; Potvin & Naud, 2011), backhauls (Belmecheri, Prins, Yalaoui, & Amodeo, 2013; Salhi, Wassan, & Hajarat, 2013), open routes (Li, Leung, & Tian, 2012), overloads (Kritikos & Ioannou, 2013), site-dependencies (Chao, Golden, & Wasil, 1999; Nag, Golden, & Assad, 1988), multi-vehicle task assignment (Franceschelli, Rosa, Seatzu, & Bullo, 2013), green routing (Juan, Goentzel, & Bektaş, 2014; Koç, Bektaş, Jabali, & Laporte, 2014), single and double container loads (Lai, Crainic, Di Francesco, & Zuddas, 2013), two-dimensional loading (Dominguez, Juan, Barrios, Faulin, & Agustin, 2014; Leung, Zhang, Zhang, Hua, & Lim, 2013), time-dependencies (Afshar-Nadjafi & Afshar-Nadjafi, 2014), multi-compartments (Wang, Ji, & Chiu, 2014), multiple stacks (Iori & Riera-Ledesma, 2015) and collection depot (Yao, Yu, Hu, Gao, & Zhang, 2015).

2.2. Mathematical formulations

We now present three formulations for the HVRP, two based on commodity flows and one based on set partitioning. The common notations of all three formulations are as follows. Each customer i has a non-negative demand q_i. Let $\mathcal{H} = \{1, \dots, k\}$ be the set of available vehicle types. Let t^h and Q_h denote the fixed vehicle cost and the capacity of vehicle of type $h \in \mathcal{H}$, respectively. Let m_h be the available number of vehicles of type h.

2.2.1. Single-commodity flow formulation

The HVRP is modeled on a complete graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \{0, \dots, n\}$ is the set of nodes, node 0 corresponds to the depot, and $\mathcal{A} = \{(i, j) : 0 \le i, j \le n, i \neq j\}$ denote the set of arcs. The customer set is $\mathcal{N}_0 = \mathcal{N} \backslash \{0\}$. Let c_{ij}^h be the travel cost on arc $(i, j) \in \mathcal{A}$ by a vehicle of type h. Furthermore, let f_{ij}^h be the amount of commodity transported on arc $(i, j) \in \mathcal{A}$ by a vehicle of type h and let the binary variable x_{ij}^h be equal to 1 if and only if a vehicle of type $h \in \mathcal{H}$ travels on arc $(i, j) \in \mathcal{A}$.

The single-commodity flow formulation of Baldacci et al. (2008) for the HVRP is as follows:

$$\text{Minimize} \quad \sum_{h\in\mathcal{H}} \sum_{j\in\mathcal{N}_0} t^h x_{0j}^h + \sum_{h\in\mathcal{H}} \sum_{(i,j)\in\mathcal{A}} c_{ij}^h x_{ij}^h \tag{2.1}$$

$$\text{subject to} \quad \sum_{j\in\mathcal{N}_0} x_{0j}^h \le m_h \qquad h \in \mathcal{H} \tag{2.2}$$

$$\sum_{h\in\mathcal{H}} \sum_{j\in\mathcal{N}} x_{ij}^h = 1 \qquad i \in \mathcal{N}_0 \tag{2.3}$$

$$\sum_{h\in\mathcal{H}} \sum_{i\in\mathcal{N}} x_{ij}^h = 1 \qquad j \in \mathcal{N}_0 \tag{2.4}$$

$$\sum_{h\in\mathcal{H}} \sum_{j\in\mathcal{N}} f_{ji}^h - \sum_{h\in\mathcal{H}} \sum_{j\in\mathcal{N}} f_{ij}^h = q_i \qquad i \in \mathcal{N}_0 \tag{2.5}$$

$$q_j x_{ij}^h \le f_{ij}^h \le (Q_h - q_i) x_{ij}^h \qquad (i, j) \in \mathcal{A}, h \in \mathcal{H} \tag{2.6}$$

$$x_{ij}^h \in \{0, 1\} \qquad (i, j) \in \mathcal{A}, h \in \mathcal{H} \tag{2.7}$$

[1] Traditionally, the Fleet Size and Mix Vehicle Routing Problem has been abbreviated as FSMVRP, and its counterpart with time windows as FSMVRPTW. A similar convention has been adopted for the Heterogeneous Fixed Fleet Vehicle Routing Problem, by using HFFVRP and HFFVRPTW to denote its versions without and with time windows, respectively. In our view, some of these abbreviations are excessively long and defy the purpose of using shorthand notation. Hence we introduce shorter and simpler abbreviations in this paper.