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Discrete Optimization

A priori optimization with recourse for the vehicle routing problem with hard time windows and stochastic service times

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ABSTRACT

The vehicle routing problem with hard time windows and stochastic service times (VRPTW-ST) introduced by Errico et al. (2013) in the form of a chance-constrained model mainly differs from other vehicle routing problems with stochastic service or travel times considered in the literature by the presence of *hard* time windows. This makes the problem extremely challenging. In this paper, we model the VRPTW-ST as a two-stage stochastic program and define two recourse policies to recover operations feasibility when the first stage plan turns out to be infeasible. We formulate the VRPTW-ST as a set partitioning problem and solve it by exact branch-cut-and-price algorithms. Specifically, we developed efficient labeling algorithms by suitably choosing label components, determining extension functions, and developing lower and upper bounds on partial route reduced cost to be used in the column generation step. Results on benchmark data show that our methods are able to solve instances with up to 50 customers for both recourse policies.

1. Introduction

In this paper we consider the vehicle routing problem with hard time windows and stochastic service times (VRPTW-ST) that was introduced in Errico, Desaulniers, Gendreau, Rei, and Rousseau (2013). A hard time window allows us to arrive earlier than its lower bound and wait but forbids to arrive later than its upper bound. The VRPTW-ST appears in various practical applications where the drivers (e.g., technicians or repairmen) of the vehicles perform specific services at the visited customers. The central issue of the problem is to determine vehicle routes that respect the customer's time windows when the details of the service to be performed at each customer are unknown beforehand, yielding stochastic service times. Originally, Errico et al. (2013) proposed to formulate the VRPTW-ST as a chance-constrained optimization model that includes a probabilistic constraint. Specifically, this constraint ensures that the probability that the planned routes cannot meet the time windows when the service times are observed does not exceed a prefixed threshold. Although this first model enables a manager to analyze the impact of stochastic service times on the vehicle routes, it does not take into account what is actually done in the case where the planned routes turn out to be infeasible during the operations. In many cases, corrective actions entailing extra costs are applied to retrieve route feasibility. Considering how costly these corrective actions can be, it then becomes an important aspect of the problem considered.

In this paper, we propose to study the VRPTW-ST as an a priori optimization problem that involves two stages. In the first stage when service times are unknown, an *a priori plan* composed of a set of planned routes (also called the *first-stage decisions*), is computed. In the second stage, all data are revealed and the a priori plan is modified according to a given *recourse* policy. In this setting, the VRPTW-ST is modeled as a two-stage stochastic program in which the objective is to minimize the total expected cost that includes expected travel costs and costs incurred by the second stage recourse actions.

A variety of stochastic vehicle routing problems have been addressed previously. The most common sources of uncertainty encountered are demand volumes (see e.g. Bertsimas, 1992; Laporte, Louveaux, & Hamme, 2002), customers presence (see e.g. Gendreau, Laporte, & Séguin, 1995), and stochastic travel and service times. The papers dealing with stochastic travel and service times have

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exploited different modeling frameworks such as dynamic programming, chance-constrained programming and robust optimization. Laporte, Louveaux, and Mercure (1992) address a non-capacitated vehicle routing problem (VRP) with stochastic travel and service times and a soft constraint on route duration. The authors present one chance-constrained model and two models with recourse, where the recourse action consists in the payment of a penalty. Lambert, Laporte, and Louveaux (1993) present a variant of the same problem where customer time windows must be met in all scenarios. Kenyon and Morton (2003) study a problem setting similar to that of Laporte et al. (1992) but with different objective functions: one model minimizes the expected completion time, the other the probability to violate a given completion deadline. In Wang and Regan (2001), a non-capacitated VRP with soft time windows and stochastic travel times is considered where the service is skipped (but not the visit itself) when a customer is visited after its deadline. To evaluate the expected load of a route, the authors propose an algorithm of exponential complexity with respect to the number of customers. A heuristic approach combining robust optimization and stochastic programming for a particular version of the non-capacitated VRP with soft time windows, a route duration constraint, and uncertainty in the customer presence and service times is proposed in Sungur, Ren, Ordóñez, Dessouky, and Zhong (2010). Heuristic approaches for versions of the capacitated VRP with soft time windows, a soft duration limit, and stochastic travel and service times are proposed in Li, Tian, and Leung (2010) and Lei, Laporte, and Guo (2012). The recourse action considered in these papers consists in the simple payment of a penalty. A similar setting is adopted in Taş, Dellaert, van Woensel, and de Kok (2013) where a three-phase tabu search algorithm is developed to minimize a combination of expected earliness and lateness at customers, and generalized operational costs. A robust optimization approach for the VRP with travel and demand uncertainty, and customer deadlines is presented in Lee, Lee, and Park (2012). Jula, Dessouky, and Ionnou (2006) and Chang, Wan, and Ooi (2009) develop heuristics for the traveling salesman problem (TSP) with hard time windows and stochastic travel and service times. Other authors considered sources of uncertainty different from service or travel times in combination with hard time window constraints. Erera, Savelsbergh, and Uyar (2009) propose a two-stage approach for the VRP with stochastic demands, a capacity constraint, and hard time windows. Campbell and Thomas (2008) investigate the TSP with customer deadlines and uncertainty in the presence of customers.

In the context of the VRP, exact solution methods addressing a combination of service time uncertainty and hard time windows have been considered for the first time in Errico et al. (2013). With respect to the latter, the present work states the VRPTW-ST as a two-stage stochastic program, including also the definition of two new recourse strategies. In fact, recourses consisting only in the payment of a penalty, as common in the vehicle routing literature with stochastic times, are not applicable in our case because time windows are hard and no late service is allowed. Adopting a two-stage stochastic model gives rise to a very challenging problem requiring the development of new solution methods.

This paper makes several contributions toward designing exact algorithms to solve recourse-based models for the VRPTW-ST. The two problem variants studied (one for each recourse policy) are formulated as set partitioning models and the procedure of Errico et al. (2013) is adopted to compute the probability that a route is feasible. To solve the proposed models, we implement state-of-the-art branch-price-and-cut algorithms. The major methodological challenges are encountered in the solution of the column generation subproblems. We devise label-setting algorithms in which labels have components to properly account for the reduced costs that depend on the service time probability distributions. Furthermore, we show how to compute the expected cost of a complete route and of a partial route and derive from these computations label extension functions. We also propose several upper and lower bounds that are used to define efficient dominance rules. These techniques are all dependent on the recourse policy used and ad hoc developments are necessary. An extensive computational study is performed using the instance set proposed in Errico et al. (2013). The results show that our methods are able to efficiently solve instances with up to 50 customers for both recourse policies.

The reminder of the paper is organized as follows. In Section 2, we formally describe the problem, including the two recourse policies, and formulate its two variants as set partitioning models. In addition, we show how to compute the expected cost of a route. In Section 3, we detail our branch-price-and-cut algorithms. The results of our computational study are reported in Section 4. Finally, we conclude in Section 5.

2. The VRPTW-ST

We first provide a general statement of the VRPTW-ST in Section 2.1 and introduce the recourse policies in Section 2.2. Then we present a generic set partitioning model for the VRPTW-ST in Section 2.3. This model is adapted afterwards for each policy by specifying the feasible route sets (Section 2.4), and how to compute the probability that a route is operationally feasible (Section 2.5) and the route expected costs (Section 2.6).

2.1. Problem statement

Consider a directed graph $G = (V, A)$, where $V = \{0, 1, \ldots, n\}$ is the node set and $A = \{(i, j)|i, j \in V\}$ the arc set. Node 0 represents a depot where a fleet of homogeneous vehicles is initially located, and $V_c = \{1, \ldots, n\}$ is the customer set. A time window $[a_i, b_i]$ and a stochastic service time are associated with each customer $i \in V_c$. In particular, service times are uncertain at the planning stage (i.e., the first stage of the problem), but their probability distributions are assumed to be known and mutually independent. The actual value of a service time is only observed once a vehicle arrives at the associated customer's location. We assume that a fixed time t^{eval} (independent of the customer) is needed to evaluate this service time. This evaluation must start within the customer's time window and its time is not part of the service time. Without loss of generality, we associate with node 0 an unconstraining time window $[a_0, b_0]$ and a constant service time s_0 equal to 0 (seen as a stochastic service time with a single possible value). Denote by t_i^{eval} the time required to evaluate the service time at node $i \in V$, where $t_i^{eval} = t^{eval}$ if $i \in V_c$ and $t_0^{eval} = 0$. A non-negative travel cost c_{ij} and a travel time t_{ij} are associated with each arc $(i, j) \in A$. Furthermore, a "no-service" penalty π is given.

An a priori plan for the VRPTW-ST is a set of a priori vehicle routes such that each route starts and ends at node 0 and all the customers are assigned to exactly one route. An a priori route is said *operationally feasible* (op-feasible, for short) if, at every customer visited along this route, the evaluation of the service time starts within the customer's time window and service is performed. However, vehicles can arrive at customers before the beginning of their time window. In this case the beginning of the service time evaluation must be postponed until the time window opens. Vehicles are not allowed to arrive after the end of a time window. It should be observed that the revealed service times directly determine if a route is op-feasible. Whenever the actual service times induce a route to be op-infeasible, recourse actions have to be taken to regain feasibility. For the models proposed in this paper, we define recourse actions that are based on skipping given customers along the route. The choice of which customer to skip is determined by the particular recourse policy. Two such policies are used to define the proposed two-stage stochastic models. In both cases, each skipped customer entails a penalty of π that may represent the dissatisfaction of a customer whose service is