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Discrete Optimization

Mathematical formulations and exact algorithm for the multitrip cumulative capacitated single-vehicle routing problem

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ABSTRACT

This paper addresses the multitrip Cumulative Capacitated Single-Vehicle Routing Problem (mt-CCSVRP). In this problem inspired by disaster logistics, a single vehicle can perform successive trips to serve a set of affected sites and minimize an emergency criterion, the sum of arrival times. Two mixed integer linear programs, a flow-based model and a set partitioning model, are proposed for small instances with 20 sites. An exact algorithm for larger cases transforms the mt-CCSVRP into a resource-constrained shortest path problem where each node corresponds to one trip and the sites to visit become resources. The resulting problem can be solved via an adaptation of Bellman–Ford algorithm to a directed acyclic graph with resource constraints and a cumulative objective function. Seven dominance rules, two upper bounds and five lower bounds speed up the procedure. Computational results on instances derived from classical benchmark problems for the capacitated VRP indicate that the exact algorithm outperforms a commercial MIP solver on small instances and can solve cases with 40 sites to optimality.

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1. Introduction

A trend in the last decade is to apply operations research to facilitate logistic operations in humanitarian disasters. Several models have been proposed to cope with mitigation, preparedness, response and recovery operations, see for instance the surveys of Altay and Green III (2006) and Galindo and Batta (2013). An important issue after a disaster is to determine the transportation routes for first aids, supplies, rescue personnel or equipment from supply points to a set of destination sites, geographically scattered over the disaster region. In this context, the arrival time of relief supplies at the affected communities clearly impacts the survival rate and the suffering of victims. Hence, the total length or duration of the routes used in vehicle routing for industrial logistics must be replaced by more pertinent service-based objective functions in humanitarian logistics (Campbell, Vandenbussche, & Hermann, 2008).

The aim of this paper is to solve exactly, via a reformulation as a shortest path problem, a version of the Capacitated Vehicle Routing Problem (CVRP) inspired by the response phase of relief operations. A single vehicle with limited capacity and range is allowed to perform multiple trips and the classical objective function (total travel time or distance) becomes the sum of arrival times at affected sites. We call this problem the multitrip Cumulative Capacitated Single-Vehicle Routing Problem (mt-CCSVRP).

The paper is structured as follows. Section 2 reviews related problems. In Section 3 the problem is formally defined and two mixed integer linear models are proposed. Section 4 describes the procedure to transform the mt-CCSVRP into a resource-constrained shortest path problem in an directed acyclic graph. An ad-hoc shortest path algorithm is developed in Section 5. Computational results are presented in Section 6 for the two models and the exact method. Concluding remarks close the paper in Section 7.

2. State of the art

After a disaster, the deliveries to affected sites must take the urgency of the situation into account. Campbell et al. (2008) suggest that using service-based objective functions may better reflect the different priorities and strategic goals found in delivering humanitarian aid. The minimization of the average arrival time is more pertinent to address the emergency of humanitarian logistic operations than classical objective functions, such as the minimization of the total distance traveled. We use in this paper an equivalent objective, the sum of arrival times.

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The sum of arrival times has been already used in some routing problems. For instance, the Minimum Latency Problem is a variation of the Traveling Salesman Problem (TSP), which consists in finding a tour starting at a depot and visiting each other node only once, in such a way that the total latency is minimized (Archer & Williamson, 2003; Blum, Chalasani, Coppersmith, Pulleyblank, Raghavan, & Sudan, 1994). The latency of a node is defined as the total distance or travel time to reach it. This problem is also known as the Delivery Man Problem (Fischetti, Laporte, & Martello, 1993) or the Traveling Repairman problem (TRP) (Jothi & Raghavachari, 2007; Tsitsiklis, 1992) because of its possible application to maintenance.

The Multiple Traveling Repairman Problem or k-TRP is a generalization of the minimum latency problem where k tours must be determined (Jothi & Raghavachari, 2007). In the Time-Dependent TSP (TDTSP), the traversal cost of an arc depends on its position in the tour (Gouveia & Voss, 1995; Lucena, 1990; Picard & Queyranne, 1978). When the objective of the TDTSP is to minimize the sum of distances traveled from the depot to each node, the problem is known as the TSP with cumulative cost or Cumulative TSP (CTSP) (Bianco, Mingozzi, & Ricciardelli, 1993).

The Cumulative Capacitated Vehicle Routing Problem (CCVRP), described by Ngueveu, Prins, and Calvo (2010), is a variant of the classical CVRP where the objective function becomes the sum of arrival times at demand nodes. The CCVRP can also be considered as the generalization of the minimum latency problem to several vehicles. These authors provide a mathematical model, several lower bounds and two memetic algorithms. Solutions are compared using classical CVRP instances from Christofides, Mingozzi, and Toth (1979), replacing the total length of the routes by the sum of arrival times, and on TRP instances from Salehipour, Sörensen, Goos, and Bräysy (2008).

The results confirm that the optimal solutions can be quite different in the CCVRP and the classical CVRP, as already observed by Campbell et al. (2008). In Euclidean versions for instance, CVRP solutions with edge crossings are suboptimal, while the elimination of these crossings often brings no improvement for the cumulative objective. More recently, Ribeiro and Laporte (2012) presented an Adaptive Large Neighborhood Search (ALNS) for the CCVRP which is compared with the memetic algorithms from Ngueveu et al. (2010), while Ke and Feng (2013) proposed a two-phase metaheuristic which improves some best known solutions from Ngueveu et al. (2010) and Ribeiro and Laporte (2012). Lysgaard and Wøhlk (2014) developed a branch and cut and price algorithm, able to solve CCVRP instances up to 69 required sites.

A comparison between the minimization of cost, maximal arrival time and average arrival time for the TSP and CVRP is given by Campbell et al. (2008). Their paper presents lower bounds, an insertion heuristic and a local search procedure. The minimization of maximal arrival time is also addressed by Applegate, Cook, Dash, and Rohe (2002) via a branch-and-cut algorithm, while Hemel, van Erk, and Jenniskens (1996) solve a practical problem aiming at minimizing the maximal tour length, using the average arrival time to break ties. Dell, Batta, and Karwan (1996) add a multi-period horizon and equity constraints.

A common assumption is that each vehicle makes a single trip, which is not realistic in disaster logistics where helicopters, for instance, can perform multiple sorties. The multitrip extension of the classical CVRP is known as the multitrip Vehicle Routing Problem (mt-VRP) and was first studied by Fleischmann (1990). Rivera, Afsar, and Prins (2015) introduce the cumulative version, called multitrip Cumulative Capacitated Vehicle Routing Problem (mt-CCVRP). They develop a flow-based model, a Multi-Start Iterated Local Search (MS-ILS) and a dominance rule for the order of routes in a multitrip. The model and the MS-ILS are evaluated on the mt-CCVRP and its particular case, the CCVRP. Rivera, Afsar, and Prins (2014) present a Multi-Start Evolutionary Local Search (MS-ELS) for the mt-CCVRP, which alternates between giant tours covering all the nodes and mt-CCVRP solutions (using a splitting procedure) and calls a variable neighborhood descent for improvement. The metaheuristic is compared with the MS-ILS of Rivera et al. (2015).

Few authors have investigated multitrip single-vehicle routing problems. Azi, Gendreau, and Potvin (2007) study a multi-trip single-vehicle routing problem with time windows (mt-SVRPTW) by enumerating all feasible trips. As it is not always possible to cover all customers, they maximize the number of visited customers and then minimize the total length. The problem is formulated as an Elementary Resource-Constrained Shortest Path Problem (ERCSPP) on an auxiliary graph and solved using a label correcting algorithm inspired by the one proposed by Feillet, Dejax, Gendreau, and Gueguen (2004).

Angel-Bello, Martínez-Salazar, and Alvarez (2013) seem to be the only authors who studied the mt-CCSVRP. They propose two mixed integer linear models for a version with a maximum number of trips. Evaluated on randomly generated instances, the best model can consistently solve instances up to 25 customers but the number of trips in the solution is limited to 2 or 3.

Our method is inspired by Azi et al. (2007) but the problems are quite different. Azi et al. consider time windows, their auxiliary graph contains circuits, and they try to visit a maximum number of sites while minimizing the total distance. In our case, there are no time windows, all sites must be visited, and the goal is to minimize the sum of arrival times at the sites. These features induce specific dominance rules and lower and upper bounds. In particular, we prove an important dominance rule which leads to an acyclic auxiliary graph.

3. Problem definition and mixed integer linear models

The mt-CCSVRP uses the sum of arrival times at sites as objective function, like in the cumulative CVRP, but involves a single vehicle which can perform more than one trip. This flexibility is necessary when the total demand exceeds the capacity of the vehicle and/or the range of the vehicle is limited. In the context of humanitarian logistics, the mt-CCSVRP models the distribution of relief supplies to a set of sites affected by a disaster, using for instance one helicopter which can do multiple sorties. After the disaster, the victims are waiting for rescue and multiple crews can be coordinated to use the vehicle full time. Nevertheless, the limited range of the vehicle imposes an upper bound on the flight time of each trip.

The problem can be defined on an complete undirected graph $G = (V, E)$. The node-set $V = \{0, \dots, n\}$ includes one depot-node 0 and a subset $V' = V \setminus \{0\}$ of n locations affected by a disaster, called *sites* because *customers* is here inappropriate. The set E is composed of edges (i, j) with travel times w_{ij} satisfying the triangle inequality, also called *flight times*. A single vehicle of capacity Q and range L_{max} (in terms of flight time) is based at the depot. Each site $i \in V'$ has a demand q_i and a service time σ_i. It is assumed without loss of generality that $\sum_{i \in V'} q_i \geq Q$ and $q_i \leq Q, \forall i \in V'$.

The objective is to identify a sequence of trips $(1, 2, \dots, \nu)$, called *multitrip*, such that each site is visited exactly once and the sum of arrival times is minimized (in the last trip, the return to the depot is not counted). A trip k is a cycle, starting and ending at the depot, whose total load W_k fits vehicle capacity Q and total flight time L_k does not exceed the range L_{max}. Each trip k has also a setup time (loading time) μ_k before leaving the depot, computed as the sum of service times over all sites served by the trip, weighted by a given coefficient β. The loading times and service times do not affect vehicle range but delay the arrivals at sites.

In our version, the number of trips ν is a decision variable. Contrary to the mt-CVRP, the cumulative objective is sensitive to the trip ordering: Theorem 2 in Section 4 defines the optimal ordering for a given set of trips. Fig. 1 shows a small instance with $n = 5$, $Q = 20$, all $q_i = 10$, all $\sigma_i = 0$, and a three-trip feasible solution. Dashed arcs correspond to transitions between two successive trips, called

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