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Decision Support

A new proposal for fusing individual preference orderings by rank-ordered agents: A generalization of the Yager's algorithm

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ABSTRACT

The problem of the aggregation of multi-agents preference orderings has received considerable attention in the scientific literature, because of its importance for different fields of research. Yager (2001) proposed an algorithm for addressing this problem when the agents' importance is expressed through a rank-ordering, instead of a set of weights. The algorithm by Yager is simple and automatable but is subject to some constraints, which may limit its range of application: (i) preference orderings should not include incomparable and/or omitted alternatives, and (ii) the fused ordering may sometimes not reflect the majority of the multi-agent preference orderings.

The aim of this article is to present a generalized version of the algorithm by Yager, which overcomes the above limitations and, in general, is adaptable to less stringent input data. A detailed description of the new algorithm is supported by practical examples.

1. Introduction

A general problem, which may concern practical contexts of different nature, is to aggregate multi-agent orderings of different alternatives into a single fused ordering. Considering the example in Table 1, *M* decision-making agents[1] (D_1 to D_M) formulate preference orderings among *n* alternatives of interest (*a*, *b*, *c*, *d*, etc.). Each ordering allows statements like $a > b$, $a \sim b$, $b > a$, where symbols "$>$" and "$\sim$" respectively mean "strictly preferred to" and "indifferent to". The objective is to aggregate the *M* agents' orderings into a single fused one, which should reflect them as much as possible, even in the presence of diverging preferences. For this reason, the fused ordering can also be defined as *consensus* or *compromise* ordering (Cook, 2006; Herrera-Viedma, Cabrerizo, Kacprzyk, & Pedrycz, 2014). Aggregation should also take into account the agents' importance, which is not necessarily equal for all of them.

This decision-making problem is very diffused in a variety of real-life contexts, ranging from *multi-criteria decision aiding/making* to *social choice theory* (Kelly, 1991); as an example, Table 2 illustrates some practical applications. Two of the reasons for this diffusion are that (i) preference orderings are probably the most intuitive and effective way to represent preference judgments of alternatives, and (ii) they do not require a common reference scale – neither numeric, linguistic or ordinal – to be shared by the interacting agents (Chen, Liu, Wang, & Augusto, 2012; Yager, 2001).

The literature embraces a variety of aggregation techniques, which are relatively interchangeable among the fields of application. Despite this variety, they can generally be divided in two categories (Arrow & Rayanaud, 1986):

1. Methods in which all agents have the same importance (Zhu, 2003); e.g., let us consider the classical approaches in the *voting theory* field (Borda, 1781; Condorcet, 1785; Lepelley & Martin, 2001);
2. Methods in which agents have recognized abilities and attributes and/or privileged positions of power, represented by weights (Dubois, Godo, & Prade, 2012; Xu, 2004); e.g., let us consider the ELECTRE or the PROMETHEE methods, in the *multicriteria decision aiding/making* field (Brans & Mareschal, 2005; Figueira, Greco, & Ehrgott, 2005).

Considering the second category methods, the definition of the agents' weights is a very delicate issue. In some settings, the weight of an agent may be well defined; for example, the Gross National Product (GNP) or population size of a country represented by the member on an International committee can immediately be used as weights. In many situations the definition of the weights is controversial, because there are no indisputable criteria or substitution rates

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[1] By a decision-making agent we will consider any of a wide variety of different types of entities. Examples could be human beings, individual criteria in a multi-criteria decision process, software based intelligent agents on the Internet, etc.

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Table 1
Problem concerning the aggregation of multi-agent preference orderings into a single fused ordering.

Input			Output
Agents	Preference orderings	Importance	
D_1	$b > (a \sim c) > d$	Information on the agents' importance, which can be expressed in different forms, e.g.:	Fused ordering, which aggregates the agents' preference orderings, e.g.: $b > (a \sim c) > (d \sim e)\ldots$
D_2	$c > b > (a \sim d)$	- by a set weights ($w_1 = 0.5, w_2 = 0.2, \ldots$),	
D_3	$b > (a \sim d) > c$	- by a rank-ordering ($D_3 > D_1 > D_2 \ldots$),	
…	…	- etc.	
D_M	$d > a > b > c$		

Table 2
Examples of practical applications of the problem of interest.

Field	Agents	Alternatives	Problem description
Multicriteria decision aiding/making	Qualitative/quantitative criteria	Alternative locations	Determination of the best location where to install a new manufacturing plant on the basis of several criteria – such as road/railway infrastructure, electrical supply, labour cost, etc. (Figueira et al., 2005).
Internet	Different types of information concerning the user	Data displayed on Internet sites	Intelligent customization of data displayed on Internet sites, based on several types of information – such as user's country, websites visited previously, apps downloaded, etc. (Yager, 1997).
Quality management	Questionnaire/interview respondents	Customer requirements	Synthesis of customer requirements, which are evaluated by a sample of questionnaire/interview respondents (Griffin and Hauser, 1993; Franceschini et al., 2007).
Voting theory	Voters	Candidates in an election	Searching a reasonable mechanism for aggregating the opinions expressed by several voters on the candidates, in order to determine a winner or to rank all candidates in order of preference (Colomer, 2004).

that can be used for this operation. Weights are often imposed by decision-makers, according to political strategies (Wang, Liang, & Qian, 2014). For example, the scientific committee of a competitive examination for promotion of faculty members may decide that the scientific publications will account for 40 percent of the total performance, the International projects for 20 percent, the teaching activity for 35 percent, etc.

The literature includes several techniques about the quantification of weights. For example, the AHP procedure uses the eigenvector method to derive a weight vector relating to agents (Saaty, 1980), or the method proposed by Martel and Ben Khelifa (2000) determines the so-called "relative importance coefficient" of each agent, based on the combination of subjective and objective components.

In some settings, weights are not available or cannot be defined on *cardinal* scales. In these cases, the importance hierarchy of agents may be expressed by a rank-ordering, such as $D_1 > (D_2 \sim D_3) > \ldots > D_M$ (Yager, 2001). When the agent importance prioritization is doubtful, the formulation of orderings is certainly simpler and more intuitive than that of weights (Chen et al., 2012).

In the remainder of this paper we will focus on a specific aggregation problem in which the agents' importance is expressed through a rank-ordering. This decision-making framework can be denominated as "ordinal semi-democratic"; the adjective "semi-democratic" indicates that agents do not necessarily have the same importance, while "ordinal" indicates that their rank is defined by a crude ordering. This makes the set of the possible solutions relatively wide, since they may range between the two extreme situations of (i) *full dictatorship* – in which the resulting fused ordering coincides with the preference ordering by the most important agent (dictator) – and (ii) *full democracy* – where the agents' preference orderings are considered as equi-important.

In spite of its practicality and adaptability to a large number of practical contexts, this specific decision-making problem is almost completely ignored in the literature. Over ten years ago, Yager (2001) proposed an algorithm (hereafter abbreviated as YA, which stands for Yager's Algorithm) to address this problem in a relatively simple, fast and automatable way. Unfortunately, this algorithm has two important limitations: (i) the resulting fused ordering may sometimes not reflect the preference ordering for the majority of agents (Jianqiang, 2007) and (ii) it is only applicable to *linear* orderings, without incomparabilities and omissions of the alternatives of interest (see the example in Fig. 1(a)). These two limitations will be clarified in the next section.

The objective of this paper is to enhance the YA so as to overcome its limitations and adapt to less stringent preference orderings (e.g., like the *partial* ordering exemplified in Fig. 1(b)). The new algorithm can be interpreted as a generalization of the YA. For this reason, it will be denominated as "Generalized (Yager's) Algorithm", hereafter abbreviated as GYA.

The remainder of the paper is organized into three sections. Section 2 recalls the YA in detail, with special attention to its limitations. Section 3 illustrates the GYA, highlighting its advantages with respect to the YA. The description of both algorithms is supported by practical examples. For a structured comparison between the two algorithms, we will use a taxonomy based on four evaluation criteria (i.e., *versatility*, *consistency*, *efficiency* and *computational complexity*), defined and described in Table 3.

The concluding section summarizes the original contributions of this paper and its practical implications, limitations and suggestions for future research.

2. Basics of the Yager's Algorithm (YA)

In Section 2.1 we take the liberty to illustrate the algorithm by Yager from a "pedagogical" point of view. For a more rigorous description, we refer the reader to the original contribution by Yager (2001). Section 2.2 discusses the (dis)advantages of this algorithm, from the perspective of the criteria in Table 3.

2.1. YA description

The algorithm can be schematized in three basic phases (mentioned in Table 4) which are described individually in the next three sub-sections.