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Decision Support

A defaultable HJM modelling of the Libor rate for pricing Basis Swaps after the credit crunch

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ABSTRACT

A great deal of recent literature discusses the major anomalies that have appeared in the interest rate market following the credit crunch in August 2007. There were major consequences with regard to the development of spreads between quantities that had remained the same until then. In particular, we consider the spread that opened up between the Libor rate and the OIS rate, and the consequent empirical evidence that FRA rates can no longer be replicated using Libor spot rates due to the presence of a Basis spread between floating legs of different tenors. We develop a credit risk model for pricing Basis Swaps in a multi-curve setup. The Libor rate is considered here as a risky rate, subject to the credit risk of a generic counterparty whose credit quality is refreshed at each fixing date. A defaultable HJM methodology is used to model the term structure of the credit spread, defined through the implied default intensity of the contributing banks of the Libor corresponding to a chosen tenor. A forward credit spread volatility function depending on the entire credit spread term structure is assumed. In this context, we implement the model and obtain the price of Basis Swaps using a numerical scheme based on the Euler–Maruyama stochastic integral approximation and the Monte Carlo method.

1. Introduction

After the credit crunch of summer 2007 the interest rate market changed due to the appearance of Basis spreads between rates with different tenors, to the loss of the possibility of replicating swap with spot rates, and to the fact that the interest rate curve underlying of interest rate derivatives does not coincide with the discounting interest rate curve anymore. In Morini (2009) and Morini (2011) the author gives a deep and detailed analysis on the causes and consequences of the interest rate market changes. The author designs a new approach for modelling collateralized derivatives, namely derivatives that are not affected by both the risk of default and liquidity because they are traded with a provision for liquidity. Morini (2009) shows that the gap between the Forward Rate Agreement, FRA, rates and their standard spot Libor replication can be explained by the existence of a premium associated to tenor, expressed by quoted Basis Swap spreads. Among the major anomalies that arose in the interest rate market there is the discrepancy between Libor rates and Eonia OIS rates, Overnight Indexed Swaps rates, that leads to a new definition of the Libor rate as a risky interest rate. In fact, Eonia OIS rates according to different maturities give the risk-free term structure, because the OIS rate with a generic maturity T is seen as an average of the market expectation of the overnight futures rates until T, and those rates are considered free of credit risk. On the contrary, the Libor rate is now a risky rate whose credit risk is not referred to a specific counterparty, but a generic one whose credit quality is refreshed at each fixing date. Thus, the level of Libor is provided by the fixings and assuming homogeneity and stability of Libor counterparties (banks). The fixings are trimmed averages of contributions from a panel of the most relevant banks in the market with the highest credit quality. Among papers which propose new approaches and methodologies for building models consistent with the new interest rate market situation, we recall Mercurio (2009), Ametrano and Bianchetti (2009), Henrard (2009), Pallavicini and Tarenghi (2010), Crépey, Grbac and Nguyen (2011), Eberlein and Grbac (2013), Pallavicini and Brigo (2013), and Crépey, Grbac, Ngor, and Skovmand (2014). Mercurio (2009) extends the basic lognormal LMM (Brace, Gatarek, & Musiela, 1997; Miltersen, Sandmann, & Sondermann, 1997), by adding stochastic volatility, in order to obtain the dynamics of FRA rates and to price interest rate derivatives. Ametrano and Bianchetti (2009) illustrate a methodology for bootstrapping multiple interest rate yield curves from non-homogeneous plain vanilla instruments quoted on the market, obtaining that each curve is homogenous in the tenor of the

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underlying rate. Henrard (2009) and Pallavicini and Tarenghi (2010) propose two different frameworks to construct yield curves consistent with a multi-curve situation and derive the price of interest rate derivatives. Crépey et al. (2011) apply a defaultable HJM approach to model the term structure of multiple interest rate curves. They choose a class of non-negative multidimensional Lévy processes as driving processes combined with deterministic volatility structures, in order to obtain a flexible and efficient interest rate derivative pricing model. Eberlein and Grbac (2013) model credit risk within the LMM. They propose a rating Lévy Libor model that is arbitrage-free for defaultable forward Libor rates related to risky bonds with credit ratings. They use time-inhomogeneous Lévy processes as driving processes. Recently, Pallavicini and Brigo (2013) model multiple LIBOR and OIS based interest rate curves consistently, based only on market observables and by consistently including credit, collateral and funding effects. They develop a framework for pricing collateralized interest-rate derivatives. Crépey et al. (2014) develop a parsimonious Markovian multiple-curve model for evaluating interest rate derivatives in the post-crisis setup and they use BSDE-based numerical computations for obtaining counterparty risk and funding adjustments.

Although in this paper we develop a model for pricing Basis Swaps according to the mathematical representation of interest rate market theorized by Morini (2009) and Morini (2011), we will model the term structure of multiple interest rates in a defaultable Heath–Jorrow–Morton framework, henceforth HJM, (see Heath, Jarrow, & Morton, 1992, Bielecki & Rutkowski, 2000, and Brigo & Mercurio, 2006).

In Section 2 we describe the general setting of the model, namely assumptions about the probability space and the dynamics of the defaultable instantaneous forward rate. In Section 3 we derive a defaultable representation of the Libor rate and we develop a model for pricing collateralized derivatives, and in particular Basis Swaps. Section 4 deals with the specification of defaultable dynamics in a multi-curve HJM framework in compliance with no-arbitrage conditions. In Section 5 we illustrate the numerical algorithm used to simulate the Basis Swap model and we show and analyze the numerical results. Finally, Section 6 concludes.

2. The general setting

In this section we present the general setting on which the credit model for pricing Basis Swap is built.

We consider the instantaneous yield curve implicitly defined by the Libor rate. We model the dynamics of defaultable instantaneous forward interest rates within the HJM framework, but we extend it to consider Libor rates, that is the underlying of all interest rate derivatives, refer to different counterparties at different fixing times.

We assume a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ exists, $\overline{T}$ is assumed to be the finite time horizon and $\mathcal{F} = \mathcal{F}_{\overline{T}}$ is the σ-algebra at time $\overline{T}$. All statements and definitions are understood to be valid until the time horizon $\overline{T}$.

We denote by C^z the counterparty of a lending contract at time z, that defaults at time $\tau^z > z$. The time τ^z is a stopping time, $\tau^z : \Omega \to [0, +\infty[$, defined as the first jump time of the Cox process $N(t) = \sum_{i=1}^{\infty} \mathbf{1}_{\{\tau_i^z \leq t\}}$, that is

$$\tau^z = inf\{t \geq 0 | N(t) > 0\}.$$

When we consider N counterparties $C^1, C^2, \ldots, C^N$, $N \in \mathbb{N}$, the filtration $\mathbf{F} = (\mathcal{F}_t)_{t\geq 0}$ is divided into two subfiltrations $\mathbf{F} = \mathbf{H} \vee \mathbf{F}^\tau$, which is $\mathcal{F}_t = \mathcal{H}_t \vee F_t^\tau \ \forall t \geq 0$, and $\mathbf{F}^\tau = \mathbf{F}^{\tau^1} \vee \mathbf{F}^{\tau^2} \vee \ldots \vee \mathbf{F}^{\tau^N}$. The subfiltration $\mathbf{H} = (\mathcal{H}_t)_{t\geq 0} = (\sigma(X_s : 0 \leq s \leq t))_{t\geq 0}$ is generated by the background driving process X, that is an $\mathbb{R}^d$-valued right continuous stochastic process $X = \{X_t : 0 \leq t \leq \overline{T}\}$ with left limit. It represents the flow of all background information except default itself and $\mathcal{H} = \mathcal{H}_{\overline{T}}$ is the sub-σ-algebra at time $\overline{T}$. The generic subfiltration $\mathbf{F}^{\tau^z} = (\mathcal{F}_t^{\tau^z})_{t\geq 0} = (\sigma(\mathbf{1}_{\{\tau^z \leq s\}} : 0 \leq s \leq t))_{t\geq 0}$ is generated by the right-continuous default indicator process $\mathbf{1}_{\{\tau^z \leq t\}}$. Since obviously $\mathcal{F}_t^{\tau^z} \subset \mathcal{F}_t$, $\forall t \geq 0$, τ^z is a stopping time with respect to $\mathbf{F}$, but it is not necessarily a stopping time with respect to $\mathbf{H}$. The right-continuous stochastic process $\lambda^z(t)$ is the intensity of the Cox process. It is independent of $N(t)$, it is assumed to be adapted to $\mathbf{H}$ and follows the diffusion process

$$d\lambda^z(t) = \mu_\lambda^z(t)dt + \sigma_\lambda^z(t)dW_\lambda^z(t),$$

where $\mu_\lambda^z(t)$ is the drift of the intensity process, $\sigma_\lambda^z(t)$ is the volatility of the intensity process and W_λ^z is a standard Wiener process under the objective probability measure $\mathbb{P}$. Processes W_λ^z, $z = 1, \ldots, N$, are N independent Wiener processes.

The defaultable instantaneous forward rate, $f^z(t, T)$, $0 \leq t \leq T \leq \overline{T}$, is modeled as the sum of the risk-free instantaneous forward rate, $f(t, T)$, and the instantaneous forward credit spread $\lambda^z(t, T)$, so that we have

$$f^z(t, T) := f(t, T) + \lambda^z(t, T). \tag{1}$$

Thus the forward credit spread is obtained as difference between the two forward interest rates. If $t = T$, then we obtain the defaultable spot rate $f^z(t) := f^z(t, t) = r(t) + \lambda^z(t)$, where $r(t) := f(t, t)$ represents the risk-free spot rate and $\lambda^z(t) := \lambda^z(t, t)$ is the spot credit spread. The credit spread is referred to as the Cox intensity across maturities.

In the HJM framework the term structure of risk-free interest rates is the stochastic integral equation for the forward rate

$$f(t, T) = f(0, T) + \int_0^t \mu(v, T, \cdot)dv + \int_0^t \sigma_f(v, T, \cdot)dW(v), \tag{2}$$

where $\mu(t, T, \cdot)$ is the instantaneous forward rate drift function, $\sigma_f(t, T, \cdot)$ is the instantaneous forward rate volatility function and $W(t)$ is a standard Wiener process with respect to the objective probability measure $\mathbb{P}$. The third argument in the brackets $(t, T, \cdot)$ indicates the possible dependence of the forward rate on other path dependent quantities, such as the spot rate or the forward rate itself.

Whereas the dynamics for $\lambda^z(t, T)$ is

$$\lambda^z(t, T) = \lambda^z(0, T) + \int_0^t \mu_\lambda^z(s, T, \cdot)ds + \int_0^t \sigma_\lambda^z(s, T, \cdot)dW_\lambda^z(s). \tag{3}$$

Again, the third argument in the brackets $(t, T, \cdot)$ indicates the possible dependence of the forward rate on other path dependent quantities.

Now we apply the HJM forward rate drift restriction, that is both necessary and sufficient condition for the absence of riskless arbitrage opportunities, to the dynamics of both the risk free rate and the credit spread. So we find the following forward dynamics, respectively for the risk-free forward rate and the forward credit spread, under the risk neutral probability measure $\widetilde{\mathbb{P}}$

$$f(t, T) = f(0, T) + \int_0^t \sigma_f(v, T, \cdot) \int_v^T \sigma_f(v, s, \cdot)ds dv + \int_0^t \sigma_f(v, T, \cdot)d\widetilde{W}(v),$$

and

$$\begin{aligned}\lambda^z(t, T) &= \lambda^z(0, T) + \int_0^t \sigma_\lambda^z(v, T, \cdot) \int_v^T \sigma_\lambda^z(v, s, \cdot)ds dv \\ &+ \int_0^t \rho\left[\sigma_f(v, T, \cdot) \int_v^T \sigma_\lambda^z(v, s, \cdot)ds + \sigma_\lambda^z(v, T, \cdot) \int_v^T \sigma_f(v, s, \cdot)ds\right]dv \\ &+ \int_0^t \sigma_\lambda^z(v, T, \cdot)d\widetilde{W}_\lambda^z(v),\end{aligned} \tag{4}$$

where ρ is the correlation coefficient between the two Wiener processes $\widetilde{W}(t)$ and $\widetilde{W}_\lambda^z(t)$ under the risk neutral probability measure and that are assumed to be one-dimensional (see Chiarella, Fanelli, & Musti, 2011, for further mathematical details in calculating the expression for the stochastic differential equations).