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Discrete Optimization

The fleet size and mix location-routing problem with time windows: Formulations and a heuristic algorithm

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ABSTRACT

This paper introduces the fleet size and mix location-routing problem with time windows (FSMLRPTW) which extends the location-routing problem by considering a heterogeneous fleet and time windows. The main objective is to minimize the sum of vehicle fixed cost, depot cost and routing cost. We present mixed integer programming formulations, a family of valid inequalities and we develop a powerful hybrid evolutionary search algorithm (HESA) to solve the problem. The HESA successfully combines several metaheuristics and offers a number of new advanced efficient procedures tailored to handle heterogeneous fleet dimensioning and location decisions. We evaluate the strengths of the proposed formulations with respect to their ability to find optimal solutions. We also investigate the performance of the HESA. Extensive computational experiments on new benchmark instances have shown that the HESA is highly effective on the FSMLRPTW.

1. Introduction

The design of distribution networks is critical for most companies because it usually requires a major capital outlay. Two major types of decisions intervene in this process, namely determining the locations of depots, and designing vehicle routes supplying customers from these depots. In the classical Facility Location Problem (FLP) (Balinski, 1965), it is assumed that each customer is served individually through a direct shipment, which makes sense when customer demands are close to the vehicle capacity. However, there exist several situations where customer demands can be consolidated. In such contexts, the FLP and Vehicle Routing Problem (VRP) should be solved jointly. The idea of combining location and routing decisions was put forward more than fifty years ago (Von Boventer, 1961) and has given rise to a rich research known as the Location–Routing Problem (LRP) (see Albareda-Sambola, 2015; Drexl and Schneider, 2015; Laporte, 1988; Min, Jayaraman, and Srivastava, 1998; Nagy and Salhi, 2007; Prodhon and Prins, 2014, for reviews). Applications of the LRP arise in areas as diverse as food and drink distribution, parcel delivery and telecommunication network design. Many algorithms, mostly heuristics, have been developed for the LRP and its variations over the past fifty years, including some population metaheuristics (Duhamel, Lacomme, Prins, & Prodhon, 2010; Prins, Prodhon, & Wolfler Calvo, 2006b), neighborhood-based metaheuristics (Duhamel, Lacomme, & Prodhon, 2011; Prins, Prodhon, & Wolfler Calvo, 2006a), simulated annealing (Karaoglan, Altiparmak, Kara, & Dengiz, 2012) and adaptive large neighborhood search (ALNS) algorithms (Hemmelmayr, Cordeau, & Crainic, 2012).

According to Hoff, Andersson, Christiansen, Hasle, and Løkketangen (2010), heterogeneous fleets are more common in real-world distribution problems than homogeneous ones. The Fleet Size and Mix VRP with Time Windows (FSMVRPTW), introduced by Liu and Shen (1999), is an LRP with a single depot, a heterogeneous fleet and time windows. Many heuristics have also been developed for the FSMVRPTW (see Irnich, Schneider, & Vigo, 2014), including a two-stage construction heuristic (Liu & Shen, 1999), a sequential construction heuristic (Dullaert, Janssens, Sörensen, & Vernimmen, 2002), a multi-start parallel regret construction heuristic (Dell'Amico, Monaci, Pagani, & Vigo, 2007), a three-phase multi-restart deterministic annealing metaheuristic (Bräysy, Dullaert, Hasle, Mester, & Gendreau, 2008), a hybrid metaheuristic combining a threshold acceptance heuristic with a guided local search (Bräysy, Porkka, Dullaert, Repoussis, & Tarantilis, 2009), an adaptive memory programming algorithm (Repoussis & Tarantilis, 2010), and a genetic algorithm using a unified component based solution framework (Vidal, Crainic, Gendreau, & Prins, 2014). This problem and a number of its variations are reviewed by Baldacci, Battarra, and Vigo (2008) and Baldacci and Mingozzi (2009).

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To our knowledge, a number of studies indirectly consider heterogeneous fleets in an LRP context but without taking time windows into account (Ambrosino, Sciomachen, & Scutellà, 2009; Wu, Low, & Bai, 2002). Berger, Coullard, and Daskin (2007) only consider a distance constraint. Therefore, combining heterogeneous fleets and time windows in the LRP is done here for the first time. We believe there is methodological interest in solving the Fleet Size and Mix Location-Routing Problem with Time Windows (FSMLRPTW).

In the FSMLRPTW, one considers a fleet of vehicles with various capacities and vehicle-related costs, as well as a set of potential depots with opening costs and capacities, and a set of customers with known demands and time windows. The FSMLRPTW consists of opening a subset of depots, assigning customers to them and determining a set of routes for heterogeneous vehicles such that all vehicles start and end their routes at their depot, each customer is visited exactly once by a vehicle within a prespecified time window, and the load of each vehicle does not exceed its capacity. The objective is to minimize the total cost which is made up of three components: the depot operating cost, the fixed cost of the vehicles, and the total travel costs of the vehicles. It is assumed that these costs are scaled over the same time horizon.

The contributions of this paper are as follows. We introduce the FSMLPRTW as a new LRP variant. We develop a hybrid evolutionary search algorithm (HESA) with the introduction of several algorithmic procedures specific to the FSMLRPTW. Namely, we introduce the location-heterogeneous adaptive large neighborhood search (L-HALNS) procedure equipped with a range of several new operators as the main Education procedure within the search. We also propose an Initialization procedure to create initial solutions, and a Partition procedure for offspring solutions. Finally, we develop a new diversification scheme through the Mutation procedure of solutions.

The remainder of this paper is structured as follows. Section 2 formally defines the problem and provides integer programming formulations together with valid inequalities. Section 3 presents a detailed description of the HESA. Computational experiments are provided in Section 4, and conclusions follow in Section 5.

2. Formulations for the fleet size and mix location-routing problem with time windows

In this section, we first define the FSMLRPTW, and then present several integer programming formulations and valid inequalities to strengthen them.

2.1. Notation and problem definition

The FSMLRPTW is defined on a complete directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \mathcal{N}_0 \cup \mathcal{N}_c$ is a set of nodes in which $\mathcal{N}_0$ and $\mathcal{N}_c$ represent the potential depot and customer nodes, respectively, and $\mathcal{A} = \{(i, j) : i \in \mathcal{N}, j \in \mathcal{N}\} \setminus \{(i, j) : i \in \mathcal{N}_0, j \in \mathcal{N}_0, i \neq j\}$ is the set of arcs. Each arc $(i, j) \in \mathcal{A}$ has a nonnegative distance c_{ij}. Here, the terms distance, travel time and travel cost are used in interchangeably. Each customer $i \in \mathcal{N}_c$ has a positive demand q_i. A storage capacity D^k and a fixed opening cost g^k are associated with each potential depot $k \in \mathcal{N}_0$. The index set of vehicle types is denoted by $\mathcal{H}$. Let Q^h and f^h denote the capacity and fixed dispatch cost of a vehicle of type $h \in \mathcal{H}$. Furthermore, s_i corresponds to the service time of node $i \in \mathcal{N}_c$, which must start within the time window $[a_i, b_i]$. If a vehicle arrives at customer $i \in \mathcal{N}_c$ before time a_i, it waits until a_i to start servicing the customer.

To formulate the FSMLRPTW, we define the following decision variables. Let x_{ij}^h be equal to 1 if vehicle of type h travels directly from node i to node j and to 0 otherwise. Let y_k be equal to 1 if depot k is opened and to 0 otherwise. Let z_{ik} be equal to 1 if customer i is assigned to depot k and to 0 otherwise. Let u_{ij}^h be the total load of vehicle of type h immediately after visiting node i and traveling directly to node j. Let f_{ij} be the total load of the vehicle while traverzing arc $(i, j) \in \mathcal{A}$, which is the aggregation of the u_{ij}^h variables over $\mathcal{H}$. Let t_i^h be the time at which a vehicle of type h starts serving at node i. Let t_i be the time at which service starts at node $i \in \mathcal{N}$, which is the aggregation of the t_i^h variables.

2.2. Integer programming formulations

The integer linear programming formulation of the problem, denoted by E_1, is then:

$$(E_1)\ \text{Minimize} \sum_{k\in\mathcal{N}_0} g^k y_k + \sum_{h\in\mathcal{H}}\sum_{k\in\mathcal{N}_0}\sum_{i\in\mathcal{N}_c} f^h x_{ki}^h + \sum_{h\in\mathcal{H}}\sum_{(i,j)\in\mathcal{A}} c_{ij} x_{ij}^h \tag{1}$$

subject to

$$\sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}} x_{ij}^h = 1 \quad i \in \mathcal{N}_c \tag{2}$$

$$\sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}} x_{ji}^h = \sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}} x_{ij}^h \quad i \in \mathcal{N} \tag{3}$$

$$\sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}} u_{ji}^h - \sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}} u_{ij}^h = q_i \quad i \in \mathcal{N}_c \tag{4}$$

$$u_{ij}^h \leq Q^h x_{ij}^h \quad i \in \mathcal{N}_0, j \in \mathcal{N}, i \neq j, h \in \mathcal{H} \tag{5}$$

$$\sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}_c} u_{kj}^h = \sum_{j\in\mathcal{N}_c} z_{jk} q_j \quad k \in \mathcal{N}_0 \tag{6}$$

$$\sum_{h\in\mathcal{H}}\sum_{j\in\mathcal{N}_c} u_{jk}^h = 0 \quad k \in \mathcal{N}_0 \tag{7}$$

$$u_{ij}^h \leq (Q^h - q_i) x_{ij}^h \quad i \in \mathcal{N}_c, j \in \mathcal{N}, h \in \mathcal{H} \tag{8}$$

$$u_{ij}^h \geq q_j x_{ij}^h \quad i \in \mathcal{N}, j \in \mathcal{N}_c, h \in \mathcal{H} \tag{9}$$

$$\sum_{i\in\mathcal{N}_c} q_i z_{ik} \leq D^k y_k \quad k \in \mathcal{N}_0 \tag{10}$$

$$\sum_{k\in\mathcal{N}_0} z_{ik} = 1 \quad i \in \mathcal{N}_c \tag{11}$$

$$x_{ij}^h + \sum_{q\in\mathcal{H}, q\neq h}\ \sum_{l\in\mathcal{N}, j\neq l} x_{jl}^q \leq 1 \quad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j, h \in \mathcal{H} \tag{12}$$

$$\sum_{h\in\mathcal{H}} x_{ik}^h \leq z_{ik} \quad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{13}$$

$$\sum_{h\in\mathcal{H}} x_{ki}^h \leq z_{ik} \quad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{14}$$

$$\sum_{h\in\mathcal{H}} x_{ij}^h + z_{ik} + \sum_{m\in\mathcal{N}_0, m\neq k} z_{jm} \leq 2 \quad k \in \mathcal{N}_0, (i, j) \in \mathcal{N}_c, i \neq j \tag{15}$$

$$t_i^h - t_j^h + s_i + c_{ij} \leq M\left(1 - x_{ij}^h\right) \quad i \in \mathcal{N}, j \in \mathcal{N}_c, i \neq j, h \in \mathcal{H} \tag{16}$$

$$a_i \leq t_i^h \leq b_i \quad i \in \mathcal{N}, h \in \mathcal{H} \tag{17}$$

$$x_{ij}^h \in \{0, 1\} \quad (i, j) \in \mathcal{N}, h \in \mathcal{H} \tag{18}$$

$$z_{ik} \in \{0, 1\} \quad k \in \mathcal{N}_0, i \in \mathcal{N}_c \tag{19}$$

$$y_k \in \{0, 1\} \quad k \in \mathcal{N}_0 \tag{20}$$

$$u_{ij}^h \geq 0 \quad (i, j) \in \mathcal{N}, h \in \mathcal{H} \tag{21}$$

$$t_i^h \geq 0 \quad i \in \mathcal{N}_c, h \in \mathcal{H}. \tag{22}$$