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Decision Support

Rough multiple objective programming

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ABSTRACT

In this paper, we focused on characterizing and solving the multiple objective programming problems which have some imprecision of a vague nature in their formulation. The Rough Set Theory is only used in modeling the vague data in such problems, and our contribution in data mining process is confined only in the "post-processing stage". These new problems are called rough multiple objective programming (RMOP) problems and classified into three classes according to the place of the roughness in the problem. Also, new concepts and theorems are introduced on the lines of their crisp counterparts; e.g. rough complete solution, rough efficient set, rough weak efficient set, rough Pareto front, weighted sum problem, etc. To avoid the prolongation of this paper, only the 1st-class, where the decision set is a rough set and all the objectives are crisp functions, is investigated and discussed in details. Furthermore, a flowchart for solving the 1st-class RMOP problems is presented.

1. Introduction

Decision making is a very important and much studied application of mathematical methods in various fields of human activity. In real-world situations, decisions are nearly always made on the basis of information which, at least in part, is vague in nature. In some cases (e.g. zooming out, granular computing and system complexity reduction), vague information is used as an approximation to more precise information. In such situations, this form of approximation is convenient and sufficient for making good enough decisions. In other cases (e.g. image processing and pattern recognition) and due to the limited precision in data acquisition phase, vague information is the only form of information available to the decision maker.

Since it was pioneered by Pawlak, rough set theory (RST) (Pawlak, 1982, 1996) has become a hot topic of great interest in several fields. The capability of handling vagueness and imprecision in real-life problems has attracted researchers to use RST in many fields; one of them is the 'optimization'. Actually, most real-life problems involve (1) a process of optimizing simultaneously a collection of conflicting and competing objectives (i.e. a process of multiple objective programming (MOP)) and (2) vague or imprecise descriptions of some parts of the problem. Therefore, we usually need a suitable framework for handling this hyperdization of MOP and vagueness. For conventional MOP problem (Ehrgott, 2005; Hwang & Masud, 1979), the aim is to maximize or minimize a set of objectives over a certain decision set, both of which are precisely defined. But in many realistic situations, the available data lacke vagueness and inexactness and the decision maker may only be able to specify the objectives and/or the decision set imprecisely in a 'rough sense' using RST.

Youness (2006) was the first who applied RST to the single-objective programming (SOP) problem and proposed a new optimization problem with rough decision set and crisp objective function, called "rough single-objective programming" (RSOP) problem. He also defined two concepts for optimal solutions, namely "surely optimal" and "possibly optimal". Then after, many attempts were made to overcome the concept of rough mathematical programming. For more details see (Xu & Yao, 2009a, 2009b; Osman et al., 2011; Lu, Huang, & He, 2011; Tao & Xu, 2012; Zhang, Shi, & Gao, 2009).

Hence, for the sake of acquiring more realistic models and results of real-life MOP problems, we present a new extension of RSOP models presented in Osman et al. (2011), to the case of rough multiple objective programming (RMOP). A new framework in modeling and solving the RMOP problem is proposed without requiring any additional data.

2. Rough set theory (Pawlak, 1982, 1996; Yao, 2008; Zhang & Wu, 2001)

RST was proposed by Pawlak in the mid-1980s, and presents a new mathematical approach to imperfect (vague/imprecise) knowledge. The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently, RST

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has been proven to be an excellent mathematical tool dealing with vague and imprecise descriptions of objects. It became a crucial issue for artificial intelligence and cognitive sciences, especially in the areas of machine learning, knowledge acquisition, decision analysis, knowledge discovery from databases, expert systems, inductive reasoning and pattern recognition.

RST expresses 'imprecision' by employing a boundary region of the vague object (e.g. set, number, interval, function, etc.). If the boundary region of an object is empty it means that the object is crisp (exact); otherwise the object is rough (inexact). A nonempty boundary region of an object means that our knowledge about the object is not sufficient to define it precisely. The bigger the boundary the worse (i.e. the higher the imprecision of) the knowledge we have about the object.

Let U be a non-empty finite set of objects, called the universal set, and $E \subseteq U \times U$ be an equivalence relation on U. The ordered pair $A = (U, E)$ is called an approximation space generated by E on U. E generates a partition $U/E = \{Y_1, Y_2, \ldots, Y_m\}$ where $Y_1, Y_2, \ldots, Y_m$ are the equivalence classes of the approximation space A.

In RST, any subset $M \subseteq U$ is described by its lower and upper approximations in terms of the equivalence classes of A, as follows:

$$E_*(M) = \cup\{Y_i \in U/E | Y_i \subseteq M\}$$
$$E^*(M) = \cup\{Y_i \in U/E | Y_i \cap M \neq \phi\}$$

The sets $E_*(M)$ and $E^*(M)$ (or simply M_* and M^*) are called the lower and the upper approximations of M respectively, in the approximation space A. Therefore, $M_* \subseteq M \subseteq M^*$. The difference between the upper and the lower approximations is called the boundary of M and is denoted by $\mathrm{BN}_E(M) = M^* - M_*$ (or simply M_{BN}). The set M is crisp (exact) in A iff $M_{\mathrm{BN}} = \phi$, otherwise M is rough (inexact) in A.

In RST, each element $x \in U$ is classified as *'surely'* inside M iff $x \in M_*$ or *'may be'* (I'm not sure if it is or not) inside M iff $x \in M_{\mathrm{BN}}$; otherwise x is surely outside M. Furthermore, an element $x \in U$ is said to be *"probably inside M"*, iff $x \in M^*$. On the other hand, each equivalence class $Y \in U/E$ is classified as *'completely'* included in M iff $Y \subseteq M_*$ or *'partially'* included in M iff $Y \subseteq M_{\mathrm{BN}}$, otherwise Y is completely not included in M. Furthermore, an equivalence class $Y \in U/E$ is said to be *"possibly included in M"*, iff $Y \subseteq M^*$.

3. Rough single-objective programming (Osman et al., 2011)

Consider the following crisp SOP problem

$$\max_{x \in M} g(x) \tag{1}$$

where $g(x)$ is the objective function, and M is the feasible set of the problem. In the conventional mathematical programming problem, it is assumed that all the parts (i.e. $g(x)$ and M) are defined in a crisp sense and "*max*" is a strict imperative. However, in many practical situations it may not be reasonable to require that the feasible set or the objective function be specified in a precise crisp terms. In such situations, it is desirable to use some type of modeling that is capable of handling vagueness and imprecision in the problem. This led to the hybridization between SOP and RST to get the concept of "rough single-objective programming". RSOP problems are broadly classified according to the place of roughness into three classes as follows:

1st-Class: problems with rough feasible set and crisp objective function.
2nd-Class: problems with crisp feasible set and rough objective function.
3rd-Class: problems with rough feasible set and rough objective function.

Unlike the crisp case (where the optimal value is a single crisp value), the optimal value in RSOP, denoted by $\bar{g}$, is defined by its lower and upper bounds i.e. $\bar{g}_*$ and $\bar{g}^*$ respectively, such that $\bar{g}_* \leq \bar{g} \leq \bar{g}^*$. Therefore, in RSOP we can say that:

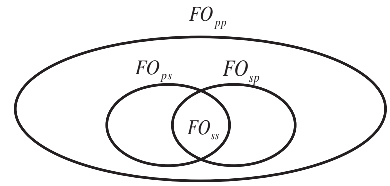

Fig. 1. The optimal sets of RSOP problem.

- a solution x is surely-optimal, if $g(x) = \bar{g}^*$,
- a solution x is probably-optimal, if $g(x) \geq \bar{g}_*$,
- a solution x is surely-not optimal, if $g(x) < \bar{g}_*$.

Also, in the 1st and 3rd classes of RSOP (where the feasible set is a rough set), it is remarkable that:

- a solution x is surely-feasible, iff it belongs to the lower approximation of the feasible set,
- a solution x is probably-feasible, iff it belongs to the upper approximation of the feasible set,
- a solution x is surely-not feasible iff it does not belong to the upper approximation of the feasible set.

Furthermore, in RSOP the optimal set is replaced by four optimal sets (See Fig. 1) covering all the possible degrees of feasibility and optimality of the solutions, as follows:

- The set of all surely-feasible, surely-optimal solutions, denoted by FO_{ss}.
- The set of all surely-feasible, probably-optimal solutions, denoted by FO_{sp}.
- The set of all probably-feasible, surely-optimal solutions, denoted by FO_{ps}.
- The set of all probably-feasible, probably-optimal solutions, denoted by FO_{pp}.

Therefore, we have:

$$\mathrm{FO}_{ss} \subseteq \mathrm{FO}_{sp} \subseteq \mathrm{FO}_{pp},\ \mathrm{FO}_{ss} \subseteq \mathrm{FO}_{ps} \subseteq \mathrm{FO}_{pp} \text{ and } \mathrm{FO}_{ss} = \mathrm{FO}_{sp} \cap \mathrm{FO}_{ps}.$$

3.1. *The 1st-class of RSOP problems (Osman et al., 2011)*

Suppose that $A = (U, E)$ is an approximation space generated by an equivalence relation E on the universe U, and $U/E = \{Y_1, Y_2, \ldots, Y_m\}$ is the partition generated by E on U. A RSOP problem of the 1st-class takes the following form:

$$\begin{aligned} &\max_{x \in M} g(x) \\ &s.t. \\ &M_* \subset M \subset M^* \\ &M_*, M^* \subseteq U/E \end{aligned} \tag{2}$$

where $g : U \to R$ is a crisp objective function. $M \subset U$ is a rough set in the approximation space A, representing the feasible set of the problem. M is given only by its lower and upper approximations, M_* and M^* respectively, and the nonempty boundary region ($M_{\mathrm{BN}} = M^* - M_* \neq \phi$) of the feasible set indicates the notion of 'rough-feasibility' in problem (2). The lower and upper bounds of the optimal objective value $\bar{g}$ in problem (2), are given by

$$\bar{g}_* = \max\{a, b\}, \quad \bar{g}^* = \max\{a, c\}$$

where (assuming the existence of the solution of the following crisp problems)

$$a = \max_{x \in M_*} g(x), \quad b = \max_{\substack{Y \in U/E, \\ Y \subseteq M_{\mathrm{BN}}}} \min_{x \in Y} g(x), \quad c = \max_{x \in M_{\mathrm{BN}}} g(x)$$

Therefore, the optimal sets of problem (2) are given as follows:

$$\mathrm{FO}_{ss} = \{x \in M_* | g(x) = \bar{g}^*\}$$