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Continuous Optimization

An iterative algorithm for two level hierarchical time minimization transportation problem

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ABSTRACT

This paper discusses a two level hierarchical time minimization transportation problem, in which the whole set of source–destination links consists of two disjoint partitions namely Level-I and Level-II links. Some quantity of a homogeneous product is first shipped from sources to destinations by Level-I decision makers using only Level-I links, and on its completion the Level-II decision maker transports the remaining quantity of the product in an optimal fashion using only Level-II links. The objective is to find that feasible solution for Level-I decision corresponding to which the optimal feasible solution for Level-II decision maker is such that the sum of shipment times in Level-I and Level-II is minimum. A polynomial time iterative algorithm is proposed to solve the two level hierarchical time minimization transportation problem. At each iteration a lexicographic optimal solution of a restricted version of a related standard time minimization transportation problem is examined to generate a pair of Level-I and Level-II shipment times and finally the global optimal solution is obtained by selecting the best out of these generated pairs. Numerical illustration is included in support of theory.

1. Introduction

An important class of optimization problems is the class of transportation problems. In a classical transportation problem, one may be interested in finding least expensive way of transporting large quantities of a homogeneous product from a number of sources to a number of destinations. Mathematically, a classical cost minimization transportation problem (CMTP) can be stated as:

$$\min_{X=\{x_{ij}\}\in S} \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}$$

where S is defined as

$$S = \left\{ X=\{x_{ij}\}\in R^{mn} \;\middle|\; \begin{array}{l} \sum_{j\in J} x_{ij}=a_i,\ \forall\, i\in I \\ \sum_{i\in I} x_{ij}=b_j,\ \forall\, j\in J \\ x_{ij}\ge 0\,\forall i\in I, j\in J \end{array} \right\} \tag{1}$$

Here, $I=\{1,2,\ldots,m\}$ is the index set of m sources, $J=\{1,2,\ldots,n\}$ is the index set of n destinations. a_i is the availability of the homogeneous product at ith source, b_j is the availability of the same at the jth destination. x_{ij} is the number of units of the homogeneous product transported from ith source to jth destination and c_{ij} is the cost of transporting one unit of the product from ith source to jth destination. This problem is called Balanced Transportation Problem if total availability at the sources is equal to the total demand at the destinations, i.e., $\Sigma_{i\in I}a_i=\Sigma_{j\in J}b_j$.

The first formulation of transportation problems dates back to the late 1930s and early 1940s. Kantorovich (1960) contributed to this field in 1939. Later on Hitchcock (1941) discussed the problem of distribution of a product from several warehouses (or sources) to numerous locations (or destinations). The problem of optimal routing of messages in a communication network, the contract award problem and routing of aircrafts and ships are important applications of transportation problems.

In a classical transportation problem, if capacity of each source–destination link is also introduced, then it is called capacitated cost minimization transportation problem (CCMTP). CCMTP is a generalization of a classical transportation problem. Mathematically, CCMTP is defined as

$$\min_{X=\{x_{ij}\}\in S} \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij}$$

along with the capacity constraint $l_{ij}\le x_{ij}\le u_{ij}$, $\forall (i,j)\in I\times J$. Here, l_{ij} and u_{ij} are the minimum and the maximum capacities of (i,j)th link respectively.

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Wagner (1959) gave an algorithm for a class of CCMTP. Extensive work has been done on capacitated CMTPs. Ford and Fulkerson (1957), Bit, Bisal, and Alam (1993), Arora and Puri (2001), Zheng, Moxu, and Min Hu (1994), Rachev and Olkin (1999), Dahiya and Verma (2007) have studied CCMTPs and its variants. Appa (1973) studied some useful variants of the CMTP. Koopmans (1947) also contributed in this field. The CMTP with mixed constraints has been studied by Brigden (1974), Klingman and Russel (1974). Khanna, Bakhsi, and Puri (1983) and Khanna and Puri (1984) presented a study on a flow constrained cost minimizing transportation problem.

Kuno and Utsunomiya (1997) proposed a pseudo-polynomial algorithm to find the global minimizer of a production-transportation problem by solving a corresponding Hitchcock transportation problem. Later on, they proposed a Lagrangian based branch-and-bound algorithm for the production-transportation problem (Kuno & Utsunomiya, 2000). Nagai and Kuno (2005) developed a branch-and-bound algorithm to find a global optimal solution of nonconvex minimization network flow problem consisting of production and transportation simultaneously.

Klanšek and Pšunder (2010) studied suitability of branch and reduce method, the branch and cut method and the combination of global and local search strategies to find the exact optimum solution of the nonlinear transportation problem. Xie and Jia (2012) developed a hybrid genetic algorithm to find an optimal solution of a nonlinear fixed charge transportation problem where the objective is to ship the available amounts of goods to satisfy the demands at the minimal total cost, on the condition that any route has a fixed cost irrelative to its shipment amount and a variable cost directly proportional to the quadratic of its shipping amount. Mizutani and Yamashita (2013) developed an algorithm to find a global optimizer of a concave cost transportation problem where the cost function is quadratic or square-root concave. Schmitzer and Schnörr (2013) presented an extension of the auction algorithm to solve linear assignment and mass transportation problem.

Various strongly polynomial time algorithms are available to solve CMTP (see, Tardos, 1985, 1986). An algorithm is strongly polynomial if it consists of (elementary) arithmetic operations and number of such operations is polynomially bounded in the dimension of the input (i.e., number of data items in the input). The best strongly polynomial running time algorithm for CMTP is $O(\overline{m+n}+mn\log(mn))$ which is obtained by Orlin (1993). This bound is obtained for uncapacitated CMTP. Kleinschmidt and Schannath (1995) developed an algorithm which runs in time proportional to $m\log m(k+n\log n)$, where k is the number of feasible arcs. This complexity bound is slightly better than the bound obtained by Orlin (1993). Later on, Sharma and Sharma (2000) proposed a computationally attractive $O(c(m+n)^2)$ dual based heuristic procedure for solving a CMTP where c is a constant.

An important class of transportation problems in terms of its prevalent applications is the bottleneck transportation problem or the time minimization transportation problem (TMTP), which usually arises in connection with the transportation of perishable commodities like fruits and vegetables, where a delay in transportation may result in much larger loss than any cost advantage attained by transporting at lower cost or situations arising in military operations where in times of emergencies the time of transportation of various military troops to the battle field is of prime importance. In this problem, a transportation time is associated between each source and each destination. In a TMTP, it is required to find a feasible distribution (of the supplies) which minimizes the total time, measured as the time of the slowest link. This problem was first addressed by Hammer (1969). The mathematical structure of this problem is as follows:

$$\min_{X\in S}\left[\max_{I\times J}(t_{ij}(x_{ij}))=T(X)\right]$$

where

$$t_{ij}(x_{ij})=\begin{cases} t_{ij}(\geq 0) & \text{if } x_{ij}>0,\\ 0 & \text{if } x_{ij}=0.\end{cases}$$

is the shipment time from the source i to the destination j.

Frieze (1975) discussed bottleneck linear programming problem which is a generalization of bottleneck transportation problem studied by Hammer (1969) and developed an algorithm to solve the problem. Mathematically, it can be stated as

$$\text{Minimize } z=\max_{\{j\,|\,x_j>0\}} c_j$$

subject to

$$Ax=b,\ x\geq 0$$

where A, x, b and $c=(c_1, c_2, \ldots, c_n)$ are respectively an $m\times n$ matrix, an n-vector, an m-vector and an n-vector.

Bansal and Puri (1980) proved that $T(X)$ is a concave function. Thus TMTP involves minimization of a concave function over a transportation polytope and hence it belongs to the class of concave minimization problems. Due to the concavity of the objective function, the search for an optimal solution is restricted to the set of basic feasible solutions only.

Important studies in TMTP have been made by Szwarc (1966), Garfinkel and Rao (1971), Bhatia, Swarup, and Puri (1977), Ahuja (1986), Prakash (1982), Chandra, Seth, and Saxena (1987), Issermsnn (1984), Arora and Puri (2001), Gupta (1977) and Hammer (1971). Later on, Burkard and Franz (1991), Arora and Puri (1997) studied lexicographic bottleneck problems. In a lexicographic time minimizing transportation problem one is not only interested in minimizing the transportation cost on the routes of the longest duration but also on the routes of second longest, third-longest duration and so on. Sherali (1982) studied multi-objective programs and proposed two algorithms to compute weights for lexicographic optimal solutions. Mazzola and Neebee (1993) developed an algorithm for bottleneck generalized assignment problem.

Almost all the techniques for solving TMTP involve an ordinary cost minimization transportation problem for which strongly polynomial algorithms are known to exist (see Orlin, 1993; Tardos, 1985). Therefore, it follows that a time minimization transportation problem is also solvable in strongly polynomial time.

Sonia and Puri (2004) discussed an invariants of time minimization transportation problem in the form of two level hierarchical time minimization transportation problem (HTMTP) in which all the source–destination links are grouped in two categories viz. Level-I links and Level-II links. In Level-I links, the leader can use only Level-I links for shipment of goods from the sources to destinations. On the completion of the shipment in Level-I, the follower uses Level-II links optimally to transport the left over quantity. Since the transportation time for the leader (Level-I) and follower (Level-II) is a concave function, it follows that transportation time in two level hierarchical time minimization transportation problem is a concave function and the objective is to find the feasible shipment schedule for the leader in Level-I so that the corresponding optimal schedule for the follower in Level-II is such that the overall shipment time for the two level hierarchical time minimization transportation problem is the least. The technique proposed by Sonia and Puri (2004) involves solving a related standard time minimizing transportation. Based upon the solution of this related time minimization transportation problem, a corresponding cost minimizing transportation problem is constructed whose optimal basic feasible solution (OBFS) yields the first feasible solution of two level HTMTP. A time minimization transportation problem with respect to Level-II shipment time is defined and based upon its OBFS a cost minimization transportation problem is constructed whose OBFS yields the second feasible solution of two level HTMTP. This process of defining a TMTP with respect to Level-II shipment time and constructing CMTP based upon its optimal solution is